Endogenous Markups in the New Keynesian Model: Implications for Inflation-Output Trade-Off and Optimal Policy

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Abstract

The standard new Keynesian model is unable to generate the inflation-output trade-off that Central Banks face in the real world, unless fluctuations are driven by shocks to desired price or wage markups. In this paper, I explore whether a model with endogenous markups can generate such a trade-off in response to more conventional shocks. In this setting, the elasticity of product demand, and therefore the price markup, depends on the market share of a firm. I first prove that the change in markup, when combined with the effect of the shock creating this change, does not lead to the trade-off I seek. Then I investigate the optimal policy under endogenous markup setting. I show that the flexible price markup is not affected; hence it is optimal to target the flexible price equilibrium.

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1 Introduction

When y_t^n represents the natural rate of output and y_t represents the realization of output under sticky prices, the standard New Keynesian Phillips Curve equation

$$\pi_t = \beta Et\left\{\pi_{t+1}\right\} + \kappa\tilde{y}_t$$

implies that stabilization of the output gap ($\tilde{y} = y_t - y_t^n$) also results in stabilization of inflation, called *Divine Coincidence*[1]. This means the model is unable to create the inflation output trade-off that Central Banks face. To obtain this trade-off, Galí, Gertler & Clarida (1999) use a cost push shock (exogenous changes in price or wage markups) as follows

$$\pi_t = \beta Et\left\{\pi_{t+1}\right\} + \kappa\tilde{y}_t + u_t$$

This equation shows that a shock to u_t should be confronted by opposite movements in output gap and inflation[2].

However, exogeneity of these shocks is not a plausible assumption. Galí and Blanchard (2006) use real imperfections to show how such an assumption endogenously leads to the inflation output trade-off following technology or preference shocks. They show that under the real wage rigidity, the difference between natural rate of output,and first best output (occurring when firms do not use any markup over their marginal costs) is not constant. Hence, stabilizing inflation and output gap, $(y_t - y_t^n)$, is no longer equal to stabilizing the *welfare relevant* output gap, $(y_t - y_t^{fb})$. Therefore it is no longer desirable from welfare point of view.

I use endogenous markup setting that is applied to a monetarist model by Kimball (1995) to investigate such a model can endogenously create the inflation output trade-off without changing the distance between natural and first best levels of output.

A standard new Keynesian model rests on Calvo (1983) price and/or wage staggering, where only some firms adjust their prices each period. These models also have the constant elasticity of demand assumption of Dixit and Stiglitz (1977). This implies that while adjusting their prices, firms neglect the change in the aggregate price index induced by their own pricing decisions. In the endogenous markup setting, firms take this effect into account, and they do not change their prices (so their markups) as much as they do under the alternative constant elasticity of demand assumption[3].

[1] In the AS-AD framework, this situation can be visualized by noticing the ability of monetary authority for keeping the inflation constant and the output equal to its natural level by counteracting upon the changes in AD and LRAS curves

[2] In the AS-AD framework, an exogenous change in price or wage level corresponds to the case where SRAS shifts. In response, monetary authority, by shifting the AD curve, can stabilize either price or output in the expense of letting the other variable deviate more compared to no intervention

[3] Suppose a decrease in nominal spending. Then the producers, who are able to adjust their prices, lower them and sell more with respect to rest. Kimball (1995) specification recognizes that these firms will be confronted with lower elasticity of demand, leaving them less incentive to reduce their prices. The case is analogous to an increase in spending. As a result, prices respond less to changes in nominal spending. Following this observation,

In this study I investigate whether the endogenous change in markup leads to similar implications with the exogenous change in markup. I specifically look for

1. whether such a model can generate inflation output trade-off
2. the optimal policy under this setting

Both of which, to the author's knowledge, have not been investigated yet.

I show that endogenous markup does not lead to inflation output trade-off like in the case of exogenous shocks in desired markup. The reason is the endogenous nature of such a change; to appear, it needs some other shock, and an endogenous change in markup just mitigates the effect of this shock. For instance; following a negative supply shock;

	P_t (π_t)	y_t^n	y_t	$\tilde{y}_t = y_t - y_t^n$
Flexible prices	$\upuparrows$	$\downdownarrows$		
Sticky prices with constant markup	$\uparrow$		$\downarrow$	$\uparrow$
Sticky prices with endogenous markup	$\upharpoonright$		$\downharpoonleft$	$\upharpoonright\upharpoonright$

This demonstration shows that inflation is lower and the output gap higher under the endogenous markup case when compared with the constant markup case. This could be defined as a relatively countercyclical movement[4]. However, with respect to the flexible price case, output gap still increases along with inflation, and stabilizing one of these factors also implies stabilizing the other[5].

The rest of the paper is organized as follows. Section 2 formalizes endogenous markup setting. Section 3 explains the baseline model that I accommodate both the endogenous and exogenous changes in markup, and makes the comparison of their results. Section 4 analyses welfare implications and optimal policy under endogenous markup setting.

2 The Analytics of the Endogenous Markup Setup

Following Kimball (1995), to create an endogenous markup consumption aggregate C_t is defined as

$$\int_0^1 \psi(\frac{c_t(i)}{C_t})di = 1$$

where $\psi(1) = 1$ and $\psi(x)$ is a strictly increasing and concave function for all $x_t(i) = \dfrac{c_t(i)}{C_t} \geq 0$. When θ is constant and does not depend on the value of x (the assumption of constant

Kimball and several other researchers use this setting to increase price stickiness and obtain real rigidity in their models

[4] This occurs as the short run AS curve is less steep now. Therefore we have a smaller change in inflation but a larger change in output

[5] The result continues to hold if a demand shock is used instead of a supply shock.

elasticity), $\psi(x) = x^{\theta-1/\theta}$ and it implies Dixit and Stiglitz type consumption aggregator

$$C_t = \left[\int_0^1 c_t(i)^{\theta-1/\theta} di \right]^{\theta/\theta-1}$$

The consumer problem is defined as (using $c = y$)

$$min \int_0^1 p_t(i) y_t(i) di \quad s.t. \quad 1 = \int_0^1 \psi(\frac{y_t(i)}{Y_t}) di$$

This leads to the implicit demand curve (derivations are in appendix A)

$$\psi'(\frac{y_t(i)}{Y_t}) = \psi'(1) \frac{p_t(i)}{P_t} \tag{1}$$

with elasticity

$$\theta(x(i)) = -\frac{\psi'((i))}{x(i)\psi''(x(i))} \tag{2}$$

As it can be seen this elasticity depends on the market share of a firm, $x(i)$. The markup (at least for the flexible price supplies) is defined by the Lerner formula

$$\mu(x) = \frac{\theta(x(i))}{\theta((i)) - 1} \tag{3}$$

with elasticity $\epsilon_\mu(x(i))$

Following Kimball(1995) and Woodford (2003), for an elasticity of markup with respect to market share of the firm, $x(i)$, I will only use its value at $x = 1$ and denote it by ϵ_μ^*

3 Model

The baseline model that I accommodate both the endogenous and exogenous changes in markup follows Galí & Blanchard (2006). Firms use the production function

$$Y = M^\alpha N^{1-\alpha} \quad \text{(in logs: } y = \alpha m + (1-\alpha) n) \tag{4}$$

where N is labor input and M is other non-produced input that allows for supply (technology) shocks within the model. Each good is non-storable and is sold to identical households, who consume it in the same period. Hence, consumption of each good must be equal to output. The marginal product of labor is

$$MPN = (1-\alpha) Y/N \quad \text{(in logs: } mpn = (y - n) + \log(1-\alpha))$$

The utility of consumers is

$$U(C,N) = \log(C) - \exp\{\xi\}\frac{N^{1+\phi}}{1+\phi}$$

where ξ is preference parameter. This function implies marginal rate of substitution

$$MRS = -\frac{U_n}{U_c} = -\frac{-\exp\{\xi\}N^{\phi}}{1/C} \quad \text{(in logs:} \quad mrs = c + \phi n + \xi)$$

I start with the equilibrium under the flexible prices while maintaining the assumption of imperfect competition in the goods market. Setting $c = y$ and using the equality $\mu = mpn - mrs (= w)$ for markup[6], we obtain

$$\mu = -(1+\phi)n_2. + \log(1-\alpha) - \xi$$

where '2' denotes second best (natural) level of the variable found by flexible prices. If we combine the last equation with (4), it finds

$$\mu = -(1+\phi)\frac{y_2 - \alpha m}{(1-\alpha)} + \log(1-\alpha) - \xi \tag{5}$$

which gives the second best level of output

$$y_2 = \alpha m + \frac{(1-\alpha)}{(1+\phi)}(\log(1-\alpha) - \mu - \xi))$$

Now I turn to equilibrium with sticky prices. For the firms having sticky prices, deviation in real marginal cost is reflected with a minus sign in their markups ($mc_t = -\mu_t$). Hence parallel to the equation (5)

$$mc_t = (1+\phi)\frac{y_t - \alpha m}{(1-\alpha)} - \log(1-\alpha) + \xi \tag{6}$$

I use (5) & (6), together with (3), to derive NKPC (details are in appendix B.1)

$$\pi_t = \beta Et\{\pi_{t+1}\} + \frac{\lambda}{1 + \epsilon_\mu^* \theta^* + \alpha\theta^*}(mc_t + \mu) \tag{7}$$

where

$$\lambda = \frac{(1-\varepsilon)(1-\beta\varepsilon)}{\varepsilon}$$

or in terms of outputs

$$\pi_t = \beta Et\{\pi_{t+1}\} + \frac{(1-\varepsilon)(1-\beta\varepsilon)}{\varepsilon}\frac{1}{(1 + \epsilon_\mu^* \theta^* + \alpha\theta^*)}\frac{(1+\phi)}{(1-\alpha)}(y_t - y_{t,2}) \tag{8}$$

[6] Markup is constant under flexible prices because firms are symmetric, which results in firms to move their prices proportionally. This implies that there will be no extra demand for any of their products

[7]Both equations (7) and (8) show that we end up with no trade-off; stabilization of the output gap is still equal to stabilization of inflation. Using an endogenous markup has just caused an increase in real rigidity as is used in the literature.

Instead of being endogenous, if the change in markup results from any direct exogenous effect (as it is used in the literature), then the NKPC equation could be written as (in appendix B.2)

$$\pi_t = \beta Et\left\{\pi_{t+1}\right\} + \frac{\lambda}{(1+\alpha\theta)}(mc_t + \mu) + \frac{\lambda}{(1+\alpha\theta)}(\mu_t - \mu) \tag{9}$$

or

$$\pi_t = \beta Et\left\{\pi_{t+1}\right\} + \frac{\lambda}{(1+\alpha\theta)}(mc_t + \mu_t) \tag{10}$$

implying that not only deviation from flexible price equilibrium, but also a change in the existing markup plays a distinct role. Equation (9) explicitly shows that the shock to $\mu_t - \mu$ should be either confronted with an increase in π_t, or a decrease in mc_t, which means a decrease in y_t

4 Optimal Policy

We used equation (5) to derive the second best level of output under endogenous markup setting.

$$y_2 = \alpha m + \frac{(1-\alpha)}{(1+\phi)}(\log(1-\alpha) - \mu - \xi))$$

Efficient (first best) allocation can be found by setting $\mu = 0$

$$y_1 = \alpha m + \frac{(1-\alpha)}{(1+\phi)}(\log(1-\alpha) - \xi)$$

hence

$$y_1 - y_2 = \frac{(1-\alpha)}{(1+\phi)}\mu = \delta \tag{12}$$

So the distance between first and second best outputs is constant under the endogenous markup setting with flexible prices, like the constant markup setup of Galí & Blanchard (2006). Thus, utility losses associated with deviations from efficient allocation, y_1, should remain parallel to deviation from the flexible price allocation, y_2. This implies that the optimality of the monetary rules derived for the equation (see Galí (2008))

$$\pi_t = \beta Et\left\{\pi_{t+1}\right\} + \kappa(y_t - y_{t,2})$$

[7]If $\epsilon_\mu(x)$ is not approximated by ϵ_μ^* the solution implies

$$\pi_t = \beta Et\left\{\pi_{t+1}\right\} + \frac{\lambda}{\epsilon_\mu(x)/\epsilon_\mu^* + \epsilon_\mu^*\theta^* + \alpha\theta^*}(mc_t + \mu)$$

should be still valid. The only difference is that now

$$\kappa = \frac{(1-\varepsilon)(1-\beta\varepsilon)}{\varepsilon} \frac{1}{(1+\epsilon_{\mu}^{*}\theta^{*}+\alpha\theta^{*})} \frac{(1+\phi)}{(1-\alpha)}$$

instead of the one implied by constant elasticity of demand assumption

$$\kappa = \frac{(1-\varepsilon)(1-\beta\varepsilon)}{\varepsilon} \frac{(1+\phi)}{(1-\alpha)}$$

Thus optimal policy should still target flexible price output and zero inflation.

Evaluating monetary policies under the endogenous markup setting should be equivalent to doing the same under any effect increasing price stickiness and creating real rigidity, but I abstract myself from doing so and do not go any further. However, it seems the appropriateness of any monetary policy now should benefit from less variance in inflation, but should suffer more from the variance in the output gap.

5 Conclusion

My motivation for using an endogenous markup was to create an output gap inflation trade-off with a more realistic model than the one with an exogenous shock. The purpose of using an endogenous markup in the literature is to create price stickiness, and its trade-off implications seemed at first as a missing element not yet investigated. To this end I applied both endogenous and exogenous changes in markup settings in a single macro model. However, my result suggests that endogenous markup is unable to create the desired trade-off like an exogenous shock does.

I also show that with the endogenous change in markup flexible price markup is unaffected. Hence, the model does not create welfare losses for the flexible price equilibrium, and targeting this equilibrium is still the optimal policy as it is equivalent to stabilizing the welfare relevant output gap.

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APPENDIX

A-Derivation of Demand Side Equations

$$min \int_0^1 p_t(i)y_t(i)di \quad s.t. \quad 1 = \int_0^1 \psi(\frac{y_t(i)}{Y_t})di$$

taking derivative with respect to $y_t(i)$ finds

$$p_t(i) = \lambda\psi'(\frac{y_t(i)}{Y_t})\frac{1}{Y_t}$$

where is λ the Lagrange Multiplier. By calculating this equation at $p_t(i) = P_t$ and $y_t(i) = Y_t$, we find

$$\lambda = \frac{P_t Y_t}{\psi'(1)}$$

hence the inverse demand equation for good i is defined as

$$p_t(i) = P_t\psi'(\frac{y_t(i)}{Y_t})\frac{1}{\psi'(1)} \tag{1}$$

Rearranging the elasticity of demand

$$\theta(x_t(i)) = -\frac{\partial y_t(i)/y_t(i)}{\partial p_t(i)/p_t(i)} = -\frac{p_t(i)}{y_t(i)}\frac{1}{\partial p_t(i)/\partial y_t(i)}$$

and inserting $p_t(i)$ and $\partial p_t(i)/\partial y_t(i)$ into it

$$\theta(x_t(i)) = -\frac{1}{y_t(i)}\frac{P_t\psi'(\frac{y_t(i)}{Y_t})\frac{1}{\psi'(1)}}{P_t\psi''(\frac{y_t(i)}{Y_t})\frac{1}{Y_t}\frac{1}{\psi'(1)}} = -\frac{1}{y_t(i)/Y_t}\frac{\psi'(\frac{y_t(i)}{Y_t})}{\psi''(\frac{y_t(i)}{Y_t})} = -\frac{1}{x_t(i)}\frac{\psi'(x_t(i))}{\psi''(x_t(i))} \tag{2}$$

Finally, log linearizing around the steady state at $x_t(i) = 1$ we have the familiar demand equation for monopolistically competitive markets

$$\ln(\frac{y_t(i)}{Y_t}) = -\theta^* \ln(\frac{p_t(i)}{P_t}) \qquad where \quad \theta^* = \theta(1)$$

or

$$\frac{y_t(i)}{Y_t} = (\frac{p_t(i)}{P_t})^{-\theta^*}$$

B-Firms' Profit Maximization Problem

Maximization problem of firms is (parallel to the baseline model of Galí (2008))

$$\max_{P_t^*} \sum_{k=0}^{\infty} \varepsilon^k E_t \left\{ Q_{t,t+k}(P_t^* Y_{t+k/t} - \Psi_{t+k}(Y_{t+k/t})) \right\}$$

where $(1-\varepsilon)$ is the probability that the firm may reset its price (Calvo (1983)), $Q_{t,t+k} = \beta^k (U'(C_{t+k})/U'(C_t))(P_t/P_{t+k})$ is the stochastic discount factor, Ψ is the nominal cost function, $Y_{t+k/t}$ denotes output in period $t+k$ for a firm that last reset its price in period t, and p^* is the optimal price set by a firm at time t. The problem is is subject to demand constraints

$$Y_{t+k/t} = (\frac{P_t^*}{P_{t+k}})^{\theta^*} Y_{t+k}$$

After taking derivative, I find

$$\sum_{k=0}^{\infty} \varepsilon^k E_t \left\{ Q_{t,t+k} Y_{t+k/t}(P_t^* - \mu(x_t(i)) \Psi'_{t+k}(Y_{t+k/t})) \right\} = 0$$

The derivative of the nominal cost function by, $\Psi'_{t+k}(Y_{t+k/t})$, divided by P_{t+k} gives the real cost of marginal production. Hence, this equation, after dividing both hand side of the equality by P_{t-1}, can be written as

$$\sum_{k=0}^{\infty} \varepsilon^k E_t \left\{ Q_{t,t+k} Y_{t+k/t}(\frac{P_t^*}{P_{t-1}} - \mu(x_t(i)) MC_{t+k/t} \frac{P_{t+k}}{P_{t-1}} \right\} = 0$$

using $Q_{t,t+k} = \beta^k$ and $Y_{t+k/t} = Y$ at steady state, and applying log linearization gives

$$p_t^* - p_{t-1} = (1-\beta\varepsilon) \sum_{k=0}^{\infty} (\beta\varepsilon)^k E_t \left\{ \hat{\mu}(x_{t+k}(i)) + m\hat{c}_{t+k/t} + (p_{t+k} - p_{t-1}) \right\} \quad (11)$$

B.1: If deviation in markup is endogenous

The elasticity of markup with respect to the market share of the firm can be written as

$$\hat{\mu}(x_{t+k}(i)) = \epsilon_\mu^* (y_{t+k}(i) - y_{t+k}) = -\epsilon_\mu^* \theta^* (p^* - p_{t+k}) \quad (14)$$

The real marginal cost can be written as

$$mc_t(i) = (\omega_t - p_t) - mpn_t(i) = (\omega_t - p_t) - [y_t(i) - n_t(i) + \log(1-\alpha)]$$

If we take n_t from equation (4), the previous equation becomes

$$mc_t(i) = (\omega_t - p_t) + \alpha(y_t(i) - m_t) - \log(1-\alpha)$$

and

$$mc_{t+k/t}(i) = (\omega_{t+k} - p_{t+k}) + \alpha(y_{t+k/t}(i) - m_{t+k}) - \log(1-\alpha)$$

($\omega_{t+k} - p_{t+k}$ does not depend on the decision of firm i at time t as it depends on economy wide labor market). Hence

$$mc_{t+k/t} = mc_{t+k} + \alpha(y_{t+k/t} - y_{t+k}) = mc_{t+k} - \alpha\theta^*(p^* - p_{t+k}) \tag{15}$$

Inserting equations (14) and (15) into equation (11) and rearranging the equation finds

$$p_t^* - p_{t-1} = (1-\beta\varepsilon)\sum_{k=0}^{\infty}(\beta\varepsilon)^k E_t\left\{m\hat{c}_{t+k} - (\epsilon_\mu^*\theta^* + \alpha\theta^*)(p^* - p_{t+k}) + (p_{t+k} - p_{t-1})\right\}$$

by adding $(\pm)(\epsilon_\mu^*\theta^* + \alpha\theta^*)p_{t-1}$ to RHS of the equation, it becomes

$$p_t^* - p_{t-1} = (1-\beta\varepsilon)\sum_{k=0}^{\infty}(\beta\varepsilon)^k E_t\left\{m\hat{c}_{t+k} - (\epsilon_\mu^*\theta^* + \alpha\theta^*)(p^* - p_{t-1}) + (1 + \epsilon_\mu^*\theta^* + \alpha\theta^*)(p_{t+k} - p_{t-1})\right\}$$

now collecting the all $p_t^* - p_{t-1}$ terms at the LHS and then divide the equation by $(1+\epsilon_\mu^*\theta^* + \alpha\theta^*)$,

$$p_t^* - p_{t-1} = (1-\beta\varepsilon)\sum_{k=0}^{\infty}(\beta\varepsilon)^k E_t\left\{\frac{1}{(1+\epsilon_\mu^*\theta^* + \alpha\theta^*)}m\hat{c}_{t+k} + (p_{t+k} - p_{t-1})\right\}$$

similarly, $E_t(p_{t+1}^* - p_t)$ can be written as

$$E_t(p_{t+1}^* - p_t) = (1-\beta\varepsilon)\sum_{l=0}^{\infty}(\beta\varepsilon)^l E_t\left\{\frac{1}{(1+\epsilon_\mu^*\theta^* + \alpha\theta^*)}m\hat{c}_{t+1+l} + (p_{t+1+l} - p_t^*)\right\}$$

combining the last two equations

$$p_t^* - p_{t-1} = (1-\beta\varepsilon)\frac{1}{(1+\epsilon_\mu^*\theta^* + \alpha\theta^*)}m\hat{c}_t + \beta\varepsilon E_t(p_{t+1}^* - p_t) + (1-\varepsilon)(p_t^* - p_{t-1})$$

using $\pi_t = (1-\varepsilon)(p_t^* - p_{t-1})$ (saying that inflation is determined by the number of firms able to set their prices to the optimal level), the last equation becomes

$$\pi_t = \beta Et\left\{\pi_{t+1}\right\} + \frac{(1-\varepsilon)(1-\beta\varepsilon)}{\varepsilon}\frac{1}{(1+\epsilon_\mu^*\theta^* + \alpha\theta^*)}m\hat{c}_t \tag{7}$$

inserting equation (6) and its flexible price equivalent into equation (7) finds that

$$\pi_t = \beta Et\left\{\pi_{t+1}\right\} + \frac{(1-\varepsilon)(1-\beta\varepsilon)}{\varepsilon}\frac{1}{(1+\epsilon_\mu^*\theta^* + \alpha\theta^*)}\frac{(1+\phi)}{(1-\alpha)}(y_t - y_{t,2}) \tag{8}$$

B.2: If deviation in markup is exogenous

Equation (11) combined with equation (15) gives

$$p_t^* - p_{t-1} = (1-\beta\varepsilon)\sum_{k=0}^{\infty}(\beta\varepsilon)^k E_t\left\{\hat{\mu}_{t+k} + m\hat{c}_{t+k} - \alpha\theta(p^* - p_{t+k}) + (p_{t+k} - p_{t-1})\right\}$$

adding $(\pm)(\alpha\theta)p_{t-1}$ to RHS of the equation, it becomes

$$p_t^* - p_{t-1} = (1-\beta\varepsilon)\sum_{k=0}^{\infty}(\beta\varepsilon)^k E_t\left\{\hat{\mu}_{t+k} + m\hat{c}_{t+k} - (\alpha\theta)(p^* - p_{t-1}) + (1+\alpha\theta)(p_{t+k} - p_{t-1})\right\}$$

collecting all the $p_t^* - p_{t-1}$ terms at the LHS and dividing the equation by $(1+\alpha\theta)$, I get

$$p_t^* - p_{t-1} = (1-\beta\varepsilon)\sum_{k=0}^{\infty}(\beta\varepsilon)^k E_t\left\{\frac{1}{(1+\alpha\theta)}\hat{\mu}_{t+k} + \frac{1}{(1+\alpha\theta)}m\hat{c}_{t+k} + (p_{t+k} - p_{t-1})\right\}$$

similarly, $E_t(p_{t+1}^* - p_t)$ can be written as

$$E_t(p_{t+1}^* - p_t) = (1-\beta\varepsilon)\sum_{l=0}^{\infty}(\beta\varepsilon)^l E_t\left\{\frac{1}{(1+\alpha\theta)}\hat{\mu}_{t+1+l} + \frac{1}{(1+\alpha\theta)}m\hat{c}_{t+1+l} + (p_{t+1+l} - p_t^*)\right\}$$

Combining the last two equations

$$p_t^* - p_{t-1} = \left\{(1-\beta\varepsilon)\frac{1}{(1+\alpha\theta)}\hat{\mu}_t + (1-\beta\varepsilon)\frac{1}{(1+\alpha\theta)}m\hat{c}_t + \beta\varepsilon E_t(p_{t+1}^* - p_t) + (1-\varepsilon)(p_t^* - p_{t-1})\right\}$$

finally using $\pi_t = (1-\varepsilon)(p_t^* - p_{t-1})$, I obtain

$$\pi_t = \beta Et\left\{\pi_{t+1}\right\} + \frac{(1-\varepsilon)(1-\beta\varepsilon)}{\varepsilon}\frac{1}{(1+\alpha\theta)}m\hat{c}_t + \frac{(1-\varepsilon)(1-\beta\varepsilon)}{\varepsilon}\frac{1}{(1+\alpha\theta)}\hat{\mu}_t \tag{9}$$

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