Endogenous Markups in the New Keynesian Model: Implications for Inflation-Output Trade-Off and Welfare

November 2013

Abstract

This paper modifies the standard New Keynesian (NK) model with nominal price rigidities by using a demand side real rigidity—the endogenous markup setting—and analyzes the implications of the model for inflation-output trade-off of Central Banks. The paper also investigates the optimal monetary policy in the model. It is shown that unlike the real wage rigidity setting and also exogenous markup shocks, the endogenous markup setting cannot improve the NK models to produce the inflation and output trade-off. Finally, it is discussed that the flexible price markup is unaffected under endogenous markup setting; hence, the optimal policy is to target the flexible price equilibrium.

JEL Codes: E32, E52
Keywords: Monetary Policy, Endogenous Markups, Inflation Output Trade-off

1 Introduction

Let $y^e_t$ represents the equilibrium level of output obtained with firms having flexible prices, $y_t$ represents the realization of output in the presence of some firms with fixed prices, and $\tilde{y} = y_t - y^e_t$ is the output gap. Using these definitions, the new Keynesian Phillips Curve (NKPC) equation in (1)

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa \tilde{y}_t$$

indicates that stabilizing the output gap also results in the stabilization of the inflation, called the divine coincidence.\footnote{In the AS-AD framework, the divine coincidence can be described as the ability of monetary authority for keeping the inflation constant and the output equal to its natural level by counteracting upon the changes in AD and LRAS curves.} Therefore, the standard NK model with nominal price rigidities is unable to create the inflation output trade-off that central banks face. The common way of obtaining this trade-off from NK models is to modify equation (1) by adding it a cost push shock (exogenous changes in price or wage markups; Gali, Gertler and Clarida, 1999) as follows

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa \tilde{y}_t + u_t$$

(2)
According to (2), cost push shocks create countercyclical changes in inflation or in the output gap; hence, the divine coincidence disappears.\(^2\) The weak side of this approach is that cost push shocks are exogenous. In an attempt to obtain inflation output policy trade-off in response to more conventional—such as the technology or preference—shocks, Blanchard and Galí (2007) uses a rigidity in real wages.\(^3\) The current paper uses another type of real rigidity, the endogenous markup setting, and asks whether it has similar implications with real (wage) rigidity for NK models.

A real rigidity is a source that makes firms unwilling to change their relative prices. The need for real rigidities in monetarist models stem from the empirical facts that the sluggishness in the real price data cannot be fully explained by nominal rigidities. For the first time Ball and Romer (1990) emphasizes this need for real rigidities. Kimball (1995) and Rotemberg (1996) are some other early examples.\(^4\) In theory, these rigidities are either on the supply side (for instance, when firm specific capital is assumed, firms would be reluctant to change their prices—and their sales—because each firm’s short run marginal cost curve is increasing in its own output)\(^5\), or on the demand side (if the elasticity of consumer demand is increasing in firms’ prices, then again firms would be reluctant to change their relative prices, which is called the endogenous markup setting). The latter setting is first applied to a monetarist model by Kimball (1995), and followed by many others such as Eichenbaum and Fisher (2004) and Klenow and Willis (2006). In this world the firms having flexible prices also consider that their decisions affect the aggregate price index in the economy, so they diverge from the constant elasticity of demand assumption of Dixit and Stiglitz (1977). For example, when a positive supply shock hits the economy, the firms that are able to lower their prices will obtain higher market share and in result will be confronted with lower elasticity of demand, leaving them less incentive to reduce their prices. Likewise, at the time of a negative supply shock, the firms that are able to increase their prices will end up with lower market share and in result will be confronted with higher elasticity of demand, leaving them less incentive to increase their prices. Consequently, the sluggishness of prices increases in the Kimball’s (1995) model. Since the change in prices is lower, and in result the change in output gap is higher under the variable elasticity of demand assumption when compared with the constant elasticity of demand assumption, this paper analyzes if this countercyclicality changes inflation-output trade-off and

\(^2\)In the AS-AD framework, cost push shocks are shifts in the SRAS curve. In response, monetary authority can stabilize either the price or output level.

\(^3\)They show that under the real wage rigidity—i.e. the sluggish adjustment of real wages—, the difference between natural rate of output and first best output—that occurs when there is no market imperfection or distortion such as nonzero markups—is not constant. Therefore, stabilizing the output gap \((\bar{y}_t - \bar{y}_t^*)\), is no longer equal to stabilizing the welfare relevant output gap \((y_t - y_t^{fb})\) and is no longer desirable from the welfare point of view.

\(^4\)See Christiano, Eichenbaum and Evans (1999), Romer and Romer (2003), Bernanke, Boivin and Eliasz (2004) for a recent evidence in favor of price sluggishness in the real data. Klenow and Kryvtsov (2005), Dhyne et al. (2005) supply micro empirical findings that indicate nominal rigidities based on the frequency of firms’ price changes are not enough to explain price sluggishness in the data.

\(^5\)For rigidities on the supply side see Gali and Gertler (1999), Woodford (2003), Altig et al. (2005).
optimal monetary policy implications of the NK models.

The method we follow is to incorporate the endogenous markup setting into the standard New Keynesian framework and then use it together with the same economic model with Blanchard and Galí (2007). The results show that the endogenous markup setting does not lead to the inflation output trade-off, also that the flexible price markup is unaffected in the endogenous markup setting; hence, the optimal monetary policy is to target the flexible price equilibrium.

The rest of the paper is organized as follows. Section 2 explains the model and search for its implications for the inflation-output trade-off of central banks. Section 3 analyzes the optimal monetary policy in this environment. Section 4 concludes.

2 The Model

The set-up of the following model is taken from Blanchard and Galí (2007) that analyzes some implications of rigidity in real wages for NK models.

2.1 Consumers

The utility function for the representative consumer is given by

\[ U(C, N) = \log(C) - \exp\{\xi\} \frac{N^{1+\phi}}{1+\phi}, \]

where \(\xi\) is preference parameter. (Time and firm-specific subscripts are suppressed until they are necessary.) This utility function is used to obtain the following intratemporal marginal rate of substitution, which is equal to the real wages

\[ MRS = -\frac{U_n}{U_c} = -\frac{-\exp\{\xi\} N^\phi}{1/C} = W \quad \text{(in logs: } mrs = w = c + \phi n + \xi). \]

The Endogenous Markup Setup. The consumption aggregate \(C_t\) is defined as

\[ \int_0^1 \psi\left(\frac{c(i)}{C}\right) di = 1, \]

where \(\psi(1) = 1\) and \(\psi(x)\) is a strictly increasing and concave function for all \(x(i) = c(i)/C \geq 0\), so \(x(i)\) is the share of the goods in the consumption basket. Note that when \(\psi(x) = x^{(\theta-1)/\theta}\), this consumption function results in Dixit-Stiglitz type consumption aggregator that assumes constant elasticity of substitution among goods. (This is shown, together with the derivations for the rest of this sub-section, in appendix A.) The current study, on the other hand, follows
Kimball’s (1995) approach and uses elasticity of substitution between goods that is not constant but increasing in firm’s price level, which lowers the incentive of firms to change their prices.

The cost minimization problem of the consumer is as follows

\[
\min \int_0^1 p(i)y(i)di \quad \text{s.t.} \quad 1 = \int_0^1 \psi(c(i)/C)di.
\]

The goods are non-storable and consumed in the same period they are produced. In a closed economy environment, this assumption implies \( c = y \). Using this equality, the solution of the above problem leads to the following implicit demand curve

\[
\psi'(\frac{y(i)}{Y}) = \psi'(1)\frac{p(i)}{P}.
\]

Finally, the elasticity of the demand for equation (4) is obtained as

\[
\theta(x(i)) = -\frac{\psi'(x(i))}{x(i)\psi''(x(i))}.
\]

Notice that unlike Dixit and Stiglitz (1977) approach, the elasticity formula in (5) is not constant but depends on the market share of a firm, \( x(i) \).

### 2.2 Firms

Monopolistically competitive firms use a Cobb-Douglas type of production function

\[
Y = M^\alpha N^{1-\alpha} \quad \text{(in logs: } y = \alpha m + (1 - \alpha)n),
\]

where \( N \) is labor input and \( M \) is non-produced input with exogenous supply. (The use \( M \) allows for supply (technology) shocks in the model, otherwise it is constant.) The marginal product of labor can be found as

\[
MPN = (1 - \alpha)Y/N \quad \text{(in logs: } mpn = (y - n) + \log(1 - \alpha)).
\]

The optimal markups can be pin down by the Lerner formula

\[
\mu(x) = \frac{\theta(x(i))}{\theta(x(i)) - 1}.
\]

This markup has an elasticity \( \epsilon_\mu(x(i)) \). Following Kimball (1995) and Woodford (2003), we approximate it around \( x = 1 \), and denote it by \( \epsilon^*_\mu \).
2.3 Baseline Scenario: The Equilibrium with Flexible Prices

When prices are flexible, the symmetry of firms implies that the market shares, and as a result markups are constant along the firms. Since labor is the only input of production with a positive cost, the markups can be calculated as: \( \mu = mpn - w \). Taking \( w \) and \( mpn \) from equations (3) and (7), we find

\[
\mu = -(1 + \phi)n_2 + \log(1 - \alpha) - \xi, \tag{9}
\]

where the subscript ‘2’ denotes the second best (natural) level of labor. It is second best because even though the prices are flexible there is still an imperfect competition in the goods market (\( \mu > 0 \)).

Combining (9) with the production function in (6) yields the following equation

\[
\mu = -(1 + \phi) \frac{y_2 - \alpha m}{(1 - \alpha)} + \log(1 - \alpha) - \xi. \tag{10}
\]

The real marginal cost of production is equal to the minus markups \( mc = -\mu = w - mpn \)

\[
mc = (1 + \phi) \frac{y_2 - \alpha m}{(1 - \alpha)} - \log(1 - \alpha) + \xi. \tag{11}
\]

Finally, rearranging (10) results in the second best level of output in (12)

\[
y_2 = am + \frac{(1 - \alpha)}{(1 + \phi)}(\log(1 - \alpha) - \mu - \xi). \tag{12}
\]

2.4 The Equilibrium with Sticky Prices

Nominal price stickiness in the model is created by using the Calvo (1983) approach, which yields the following NKPC equation (the derivations are in appendix B.1)\(^6\)

\[
\pi_t = \beta E_t \{ \pi_{t+1} \} + \frac{(1 - \varepsilon)(1 - \beta \varepsilon)}{\varepsilon} \frac{1}{[1 + \epsilon_{\mu} + \alpha \theta^\gamma/(1 - \alpha)]} (mc_t - mc), \tag{13}
\]

where \( \beta \) is the discount factor and \((1 - \varepsilon)\) is the probability that the firm may reset its price at a given period. (13) indicates that inflation is positively correlated with the deviations of real marginal cost from its flexible price equilibrium.

The real marginal cost of production \((mc_t)\) in (13) can be found by using (11) and replacing \( y_2 \) with \( y_t \)

\[
mc_t = (1 + \phi) \frac{y_t - \alpha m}{(1 - \alpha)} - \log(1 - \alpha) + \xi. \tag{14}
\]

\(^6\)If \( \epsilon_{\mu}(x) \) is not approximated around \( x = 1 \) (that is \( \epsilon_{\mu}^+ \)), the solution for NKPC is obtained as

\[
\pi_t = \beta E_t \{ \pi_{t+1} \} + \frac{\lambda}{[\epsilon_{\mu}(x)/\epsilon_{\mu}^+ + \epsilon_{\mu}^+ \theta^\gamma + \alpha \theta^\gamma/(1 - \alpha)]} (mc_t + \mu)
\]
Using (11) and (14), (13) can also be written in terms of the deviations in the output rather than the deviations in the real marginal cost

\[
\pi_t = \beta Et \left\{ \pi_{t+1} \right\} + \frac{(1-\epsilon)(1-\beta\epsilon)}{\epsilon} \frac{1}{[1 + \epsilon^t \theta^* + \alpha\theta^*/(1-\alpha)]} \frac{1 + \phi}{(1-\alpha)} (y_t - y_{t,2}). \tag{15}
\]

### 2.5 Results for the Inflation Output Trade-off

Equation (15) shows that with the endogenous markup setting the stabilization of the output gap is equal to stabilization of inflation, which points out that no trade-off is generated from the model. Next, we check whether the distance between the first and second best levels of output is constant so that stabilizing the output gap to its second best level is desirable. With this regard, Blanchard and Galí (2007) indicate that “...in contrast with the baseline NKPC model, (with the model modified by the real wage rigidity) the divine coincidence no longer holds, since stabilizing the output gap \((y - y_2)\) is no longer desirable. This is because what matters for welfare is the distance of output not from its second-best level, but from its first-best level. In contrast to the baseline model, the distance between the first- and the second-best levels of output is no longer constant, but is instead affected by the shocks...” To check if this possibility arises with our model, we use \(\mu = 0\) in (12). This is to say we combine zero markups with no (real or nominal) rigidities, leading to the first best allocation

\[ y_1 = \alpha m + \frac{(1-\alpha)}{(1+\phi)} (\log(1-\alpha) - \xi). \]

The final equation together with (12) results in

\[ y_1 - y_2 = \frac{(1-\alpha)}{(1+\phi)} \mu. \tag{16} \]

Equation (16) shows that under the endogenous markup setting the distance between the first best and the second best levels of output equals to a constant term, i.e. is unaffected from technology \((m)\) or preference \((\xi)\) shocks. Therefore, stabilizing the output gap at its second best level \((y - y_2)\) is equal to stabilizing the welfare relevant output gap \((y - y_1)\). These results indicate that no trade-off arises between inflation and output in a model with endogenous markup setting. Since the term \(1 + \epsilon^t \theta^* + \alpha\theta^*/(1-\alpha)\) in (15) is greater than the zero\(^7\), equations (13) and (15) indicate that the endogenous markup setting only causes a real rigidity, i.e. a strategic complementarity between (relative) pricing decisions of the firms. Hence, even though the change in inflation is lower and the change in output gap is higher under the endogenous markup case when compared with the constant markup case, which could be defined as a relatively countercyclical movement with respect to the case with constant elasticity of demand

\(^7\)This is because \(\epsilon^t\) is approximated around 1, and \(\theta\) and \(\alpha\) are positive terms that, respectively, define the elasticity of substitution between different goods and the labor share in production.
assumption, output gap still increases in the endogenous markup case along with the inflation. As a result, both of these measures can be both stabilized at a time unless the model is modified with exogenous shocks, which is shown below.

2.6 Exogenous Changes in Markups

If we assume that markups are not endogenous but subject to exogenous cost push shocks as suggested by Galí, Gertler and Clarida (1999), then the NKPC equation can be derived as (shown in appendix B.2)

$$
\pi_t = \beta E_t \{ \pi_{t+1} \} + \frac{\lambda}{[1 + \alpha\theta/(1 - \alpha)]} (mc_t - mc) + \frac{\lambda}{[1 + \alpha\theta/(1 - \alpha)]} (\mu_t - \mu) \quad (17)
$$

(17) shows that the term \((\mu_t - \mu)\) that represents an exogenous change in the markup is affective on the inflation in addition to the term \((mc_t - mc)\) in (13). As a result, a shock to markup can either be confronted with an increase in \(\pi_t\), or a decrease in \(mc_t\) (i.e. a decrease in \(y_t\)), and the model obtains the inflation output trade-off.

3 Optimal Monetary Policy

(16) shows that the distance between the first best and the second best output levels is constant under the endogenous markup setting. This result implies that the utility losses arise from deviations from efficient allocation \((y_1)\) remain parallel to those arise from deviations from the flexible price allocation \((y_2)\), which further requires the optimality of the monetary rules for the standard NKPC equation (see Galí, 2008)

$$
\pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa(y_t - y_{t,2})
$$

to remain valid. The only difference is that now

$$
\kappa = \frac{(1 - \varepsilon)(1 - \beta\varepsilon)}{\varepsilon} \frac{1}{[1 + e^\varphi\theta^* + \alpha\theta^*/(1 - \alpha)]} \frac{(1 + \phi)}{(1 - \alpha)},
$$

instead of the one obtained with constant elasticity of demand assumption

$$
\kappa = \frac{(1 - \varepsilon)(1 - \beta\varepsilon)}{\varepsilon} \frac{(1 + \phi)}{(1 - \alpha)}.
$$

The optimal policy should target flexible price level of output and zero inflation. Generally speaking, under the endogenous markup setting monetary policies benefit from less variance in inflation and suffer more from the variance in the output gap.
4 Conclusions

This paper questions whether the central banks dilemma for stabilizing output gap or stabilizing inflation can be obtained with a model using interaction between aggregate shocks and firms’ markups, i.e. the endogenous markup setting. The use of this setting in the literature is to create price stickiness, and its inflation output trade-off and policy implications are not investigated yet. Moreover, even though constant elasticity of demand assumption is a convenient assumption for economic modelling, the strategic complementarity in pricing decisions that arises with the endogenous markup setting proves to be a more realistic representation of the real world markets.

The paper modifies the standard New Keynesian framework with the endogenous markup setting and use it within the model studied in Blanchard and Galí (2007). The findings show that the trade-off between output gap stabilization and inflation stabilization that is confronted by the central banks is not produced by this set-up.

Finally, it is shown that the flexible price markup remains unaffected from the endogenous markup setting; hence, targeting the level of output obtained with flexible prices is equivalent to targeting the first best level of output and as a result, is the optimal monetary policy.

APPENDIX

A-Derivation of the Demand Side Equations

The consumer problem is defined as follows

\[
\min \int_0^1 p(i)y(i)di \quad s.t. \quad 1 = \int_0^1 \psi(y(i)/Y)di
\]

Putting this problem into Lagrangian form and taking derivative with respect to \(y_t(i)\) yields

\[
p(i) = \lambda \psi'(\frac{y(i)}{Y}) \frac{1}{Y}
\]

where \(\lambda\) is the Lagrange Multiplier. By calculating (18) at \(p(i) = P\) and \(y_t(i) = Y\), we find

\[
\lambda = \frac{PY}{\psi'(1)}.
\]

Using this equality in (18) results in the inverse demand equation given in the text

\[
p(i) = P \psi'(\frac{y(i)}{Y}) \frac{1}{\psi'(1)}.
\]
By definition, the elasticity of demand is

\[ \theta(x(i)) = \frac{\partial y(i)/y(i)}{\partial p(i)/p(i)} = \frac{1}{y(i) \partial p(i)/\partial y(i)} \]  

(19)

Calculating \( p_t(i) \) and \( \partial p_t(i)/\partial y_t(i) \) from (4) and using them together with (19) result in

\[ \theta(x(i)) = -\frac{1}{y(i)} \frac{P_t \psi'(y(i)/Y)}{\psi'(1)} = -\frac{1}{y(i)/Y} \frac{\psi'(y(i)/Y)}{\psi'(1)} = -\frac{1}{x_t(i) \psi'(x(i))}. \]

(5)

When we use the elasticity of demand equation in (5) to log-linearize the inverse demand equation in (4) around the steady state at \( x(i) = 1 \), we obtain the familiar demand equation for monopolistically competitive markets with constant elasticity of demand assumption

\[ \ln\left(\frac{y(i)}{Y}\right) = -\theta^* \ln\left(\frac{p(i)}{P}\right) \quad \text{where} \quad \theta^* = \theta(1), \]

(20)

which can also be written as

\[ \frac{y(i)}{Y} = \left(\frac{p(i)}{P}\right)^{-\theta^*} \]

B-Firms’ Profit Maximization Problem

The maximization problem of firms follows chapter 3 of Galí (2008), and is given as

\[ \max_{P_t^*} \sum_{k=0}^{\infty} \varepsilon^k E_t \left( Q_{t,t+k}(P_t^* Y_{t+k/t} - \Psi_{t+k}(Y_{t+k/t})) \right) \]

(21)

where \( Q_{t,t+k} = \beta^k(U'(C_{t+k})/U'(C_t))(P_t/P_{t+k}) \) is the stochastic discount factor, \( \Psi \) is the nominal cost function, \( Y_{t+k/t} \) denotes output in period \( t + k \) for a firm resetting its price last time in period \( t \), \( P^* \) is the optimal price set by a firm at time \( t \), and \( (1 - \varepsilon) \) is the probability that the firm may reset its price. The problem is subject to the following demand conditions

\[ Y_{t+k/t} = (\frac{P^*_t}{P_{t+k}})^{\theta^*} Y_{t+k} \]

(22)

Inserting this constraint into the firm maximization problem in (21) and taking derivative with respect to the optimal price \( P_t^* \) yields

\[ \sum_{k=0}^{\infty} \varepsilon^k E_t \left( Q_{t,t+k} Y_{t+k/t}(P_t^* - \mu(x_t(i)))\Psi_{t+k}(Y_{t+k/t})) \right) = 0 \]
The last equation, when divided by $P_{t-1}$, can be written as

$$\sum_{k=0}^{\infty} \epsilon^k E_t \left\{ Q_{t,t+k} Y_{t+k,t} \left( \frac{P_t}{P_{t-1}} \right)^{\alpha} \mu(x_t(i)) MC_{t+k,t} \frac{P_{t+k}}{P_{t-1}} \right\} = 0$$  

(23)

where $MC$ is the real cost of marginal production and equal to the derivative of the nominal cost function, $\Psi'_{t+k}(Y_{t+k,t})$, with respect to $P_{t+k}$. At the zero inflation steady state the following conditions must hold: $P_{t-1} = P_t = P_{t+k}$, $Y_{t+k,t} = Y$, $MC_{t+k,t} = MC = 1/\mu$ and $Q_{t,t+k} = \beta^k$. Applying log linearization to (23) around this steady state finds

$$p_t^* - p_{t-1} = (1 - \beta \epsilon) \sum_{k=0}^{\infty} (\beta \epsilon)^k E_t \left\{ \hat{\mu}(x_{t+k}(i)) + m\hat{c}_{t+k,t} + (p_{t+k} - p_{t-1}) \right\}$$  

(24)

B.1: Endogenous Markups

This section uses (24) is used to derive NKPC equation in (13). First, we start with $\hat{\mu}(x_{t+k}(i))$ term in (24), which is the deviation of the firm $i$’s markup from its steady state, and it can be written in terms of the demand elasticity of markups that is approximated around $x = 1$

$$\hat{\mu}(x_{t+k}(i)) = \epsilon^*_\mu (y_{t+k}(i) - y_{t+k}).$$

Combining the previous equation with the demand equation in (22) yields

$$\hat{\mu}(x_{t+k}(i)) = -\epsilon^*_\mu \theta^* (p_t^* - p_{t+k})$$  

(25)

To derive the $m\hat{c}_{t+k,t}$ term in (24), the real marginal cost can be written as

$$mc_t(i) = w_t - m\text{pn}_t(i) = w_t - [y_t(i) - n_t(i) + \log(1 - \alpha)],$$

where $w$ denotes the real wages that depends on the economy wide labor market and the $\text{mpn}$ denotes the marginal product of labor that is taken from (7). When $n_t$ is taken from the production function in (6), the previous equation becomes

$$mc_t(i) = w_t + \frac{\alpha}{1 - \alpha} (y_t(i) - m_t) - \log(1 - \alpha),$$

so that

$$mc_{t+k,t}(i) = w_{t+k} + \frac{\alpha}{1 - \alpha} (y_{t+k,t}(i) - m_{t+k}) - \log(1 - \alpha).$$

Combining the previous two equations we obtain

$$mc_{t+k,t} = mc_{t+k} + \frac{\alpha}{1 - \alpha} (y_{t+k,t} - y_{t+k}).$$
using (20) with this equation yields

$$mc_{t+k} = mc_{t+k} - \frac{\alpha}{1-\alpha}\theta^*(p^* - p_{t+k})$$

(26)

Inserting equations (25) and (26) into (24) finds

$$p^*_t - p_{t-1} = (1 - \beta \varepsilon) \sum_{k=0}^{\infty} (\beta \varepsilon)^k E_t \{mc_{t+k} - [1 + \epsilon^*_\mu \theta^* + \alpha \theta^*/(1 - \alpha)](p^* - p_{t+k}) + (p_{t+k} - p_{t-1})\}$$

adding $(\pm)[\epsilon^*_\mu \theta^* + \alpha \theta^*/(1 - \alpha)]p_{t-1}$ to the RHS (right hand side) of this equation we obtain

$$p^*_t - p_{t-1} = (1 - \beta \varepsilon) \sum_{k=0}^{\infty} (\beta \varepsilon)^k E_t \{mc_{t+k} - (\epsilon^*_\mu \theta^* + \alpha \theta^*/(1 - \alpha))(p^* - p_{t-1}) + [1 + \epsilon^*_\mu \theta^* + \alpha \theta^*/(1 - \alpha)](p_{t+k} - p_{t-1})\}$$

Now collecting all the $p^*_t - p_{t-1}$ terms at the LHS of the equation and then divide the equation by $[1 + \epsilon^*_\mu \theta^* + \alpha \theta^*/(1 - \alpha)]$ gives

$$p^*_t - p_{t-1} = (1 - \beta \varepsilon) \sum_{k=0}^{\infty} (\beta \varepsilon)^k E_t \left\{\frac{1}{1 + \epsilon^*_\mu \theta^* + \alpha \theta^*/(1 - \alpha)} mc_{t+k} + (p_{t+k} - p_{t-1})\right\}.$$  

Similarly, $E_t(p^*_{t+1} - p_t)$ can be written as

$$E_t(p^*_{t+1} - p_t) = (1 - \beta \varepsilon) \sum_{t=0}^{\infty} (\beta \varepsilon)^t E_t \left\{\frac{1}{1 + \epsilon^*_\mu \theta^* + \alpha \theta^*/(1 - \alpha)} mc_{t+1} + (p_{t+1} - p_t)^*\right\}.$$  

Combining the last two equations

$$p^*_t - p_{t-1} = (1 - \beta \varepsilon) \frac{1}{1 + \epsilon^*_\mu \theta^* + \alpha \theta^*/(1 - \alpha)} mc_{t} + \beta \varepsilon E_t(p^*_{t+1} - p_t) + (1 - \varepsilon)(p^*_t - p_{t-1}).$$

Using $\pi_t = (1 - \varepsilon)(p^*_t - p_{t-1})$ (this equality indicates that inflation is determined by the number of firms that are able to reset their prices), the last equation becomes

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \frac{(1 - \varepsilon)(1 - \beta \varepsilon)}{\varepsilon} \frac{1}{1 + \epsilon^*_\mu \theta^* + \alpha \theta^*/(1 - \alpha)} mc_t.$$  

(13)

**B.2: Exogenous Changes in Markups**

We no longer assume that the changes in the markups are endogenous; as a result, the elasticity of demand, $\theta$, is a constant now. Calculating (26) at the constant elasticity of demand and using the result in (24) yields

$$p^*_t - p_{t-1} = (1 - \beta \varepsilon) \sum_{k=0}^{\infty} (\beta \varepsilon)^k E_t \{\mu_{t+k} + mc_{t+k} - [\alpha/(1 - \alpha)](p^* - p_{t+k}) + (p_{t+k} - p_{t-1})\}$$
adding \((\pm) [\alpha \theta / (1 - \alpha)] p_{t-1}\) to RHS of the equation, it becomes

\[
p_t^* - p_{t-1} = (1-\beta \varepsilon) \sum_{k=0}^{\infty} (\beta \varepsilon)^k E_t \left\{ \hat{\mu}_{t+k} + m_c t_{t+k} - [\alpha \theta / (1 - \alpha)] (p_t^* - p_{t-1}) + (1 + \alpha \theta / (1 - \alpha)) (p_{t+k} - p_{t-1}) \right\}
\]

collecting the \(p_t^* - p_{t-1}\) terms at the LHS of the equation and dividing the equation by \([1 + \alpha \theta / (1 - \alpha)]\), we get

\[
p_t^* - p_{t-1} = (1-\beta \varepsilon) \sum_{k=0}^{\infty} (\beta \varepsilon)^k E_t \left\{ \frac{1}{[1 + \alpha \theta / (1 - \alpha)]} \hat{\mu}_{t+k} + \frac{1}{[1 + \alpha \theta / (1 - \alpha)]} m_c t_{t+k} + (p_{t+k} - p_{t-1}) \right\}.
\]

Similarly, \(E_t(p_{t+1}^* - p_t)\) can be written as

\[
E_t(p_{t+1}^* - p_t) = (1-\beta \varepsilon) \sum_{l=0}^{\infty} (\beta \varepsilon)^l E_t \left\{ \frac{1}{[1 + \alpha \theta / (1 - \alpha)]} \hat{\mu}_{t+l+1} + \frac{1}{[1 + \alpha \theta / (1 - \alpha)]} m_c t_{t+l+1} + (p_{t+l+1} - p_t^*) \right\}.
\]

Combining the last two equations

\[
p_t^* - p_{t-1} = \left\{ \frac{(1-\beta \varepsilon)}{[1 + \alpha \theta / (1 - \alpha)]} \hat{\mu}_t + \frac{(1-\beta \varepsilon)}{[1 + \alpha \theta / (1 - \alpha)]} m_c t + \beta \varepsilon E_t (p_{t+1}^* - p_t) + (1-\varepsilon)(p_t^* - p_{t-1}) \right\}.
\]

Finally, using \(\pi_t = (1-\varepsilon)(p_t^* - p_{t-1})\) in the last equation results in

\[
\pi_t = \beta E_t \{\pi_{t+1}\} + \frac{(1-\varepsilon)(1-\beta \varepsilon)}{\varepsilon [1 + \alpha \theta / (1 - \alpha)]} m_c t + \frac{(1-\varepsilon)(1-\beta \varepsilon)}{\varepsilon [1 + \alpha \theta / (1 - \alpha)]} \hat{\mu}_t \tag{17}
\]

References


