E - Extensions of the Ramsey Growth Model

1- GOVERNMENT

- The government purchases goods and services, denoted by G, and also makes transfer payments to households in an amount V. These two forms of spending are financed by various taxes within a balanced budget

$$G + V = \tau_w wL + \tau_a r(Assets) + \tau_c C + \tau_f \cdot (firms' \; earnings)$$

- where τ_w represents the tax rate on wages
- τ_a represents the tax rate on assets owned by households
- τ_c represents the tax rate on consumption goods
- and τ_f represents the tax rate on firms' earnings
- In this setting, $-V$ can also be interpreted as a lump-sum tax

Households

- Households' budget constraint changes according to

$$\dot{a} = (1-\tau_w)w + (1-\tau_a)ra - (1+\tau_c)c - na + v$$

where small letters represent per capita terms

- The Hamiltonian is written in the following form

$$\mathtt{H} = u(c)e^{(n-\rho)t} + \lambda[(1-\tau_w)w + (1-\tau_a)ra - (1+\tau_c)c - na + v]$$

- Using the constant intertemporal elasticity of substitution utility function

$$u(c) = \frac{c^{1-\theta}-1}{1-\theta}$$

and the following FOCs

$$\frac{\partial \mathtt{H}}{\partial c} = 0 \quad \& \quad \frac{\partial \mathtt{H}}{\partial a} = -\dot{\lambda}$$

we obtain the modified Euler Equation

$$\dot{c}/c = (1/\theta)[(1-\tau_a)r - \rho]$$

where the household's decision to defer consumption depends on the after-tax rate of return, $(1-\tau_a)r$. The tax rate on consumption, τ_c, does not appear in the first-order condition because it is constant over time and affects both c_t and c_{t+1}

- The new transversality condition can be written as

$$\lim_{t\to\infty}\{a(t)\exp(-\int_0^t[(1-\tau_a)r(v)-n]dv)\} = 0$$

Firms

- They use the production function

$$Y = F(K, \hat{L})$$

where $\hat{L} = Le^{xt}$ is effective labor input

- Taxable earnings are given as

$$F(K, \hat{L}) - wL - \delta K$$

where we assume the rental payments are not tax deductible but the depreciation rate

- After-tax profits are equal to

$$(1 - \tau_f)[F(K, \hat{L}) - wL - \delta K] - rK \tag{1}$$

- Firms maximize their after-tax profits in (1) w.r.t. $\hat{k} = K/\hat{L}$, which gives

$$f'(\hat{k}) = \frac{r}{1-\tau_f} + \delta$$

  Thus, a higher τ_f raises the required marginal product of capital, $f'(\hat{k})$ (lowers the amount of $\hat{k}$) for a given r. (Remember that firms take prices as given in the perfect competition set-up)

  – As in the basic Ramsey model, we can also verify that the representative firm ends up with zero after-tax profit in equation (1)

- The firm equates the marginal product of labor to the wage rate

$$w = e^{xt}[f(\hat{k}) - \hat{k}f'(\hat{k})]$$

Equilibrium

- Using the interest rate and wages in the household and government budget constraints and the condition for equilibrium in the asset market, $\hat{a} = \hat{k}$, finds

$$\dot{\hat{k}} = f(\hat{k}) - \hat{c} - (x + n + \delta)\hat{k} - \hat{g} \tag{2}$$

  which is the resource constraint for the economy

- Using the condition for the interest rate together with the Euler equation gives

$$\dot{\hat{c}}/\hat{c} = (1/\theta)\{(1 - \tau_a)(1 - \tau_f)[f'(\hat{k}) - \delta] - \rho - \theta x\} \tag{3}$$

  so income on capital is effectively "double taxed"—through the firms earnings by τ_f and through the households incomes by τ_a

- The transversality condition now incorporates the effect of taxation

$$\lim_{t\to\infty}\{\hat{k}\exp(-\int_0^t [(1 - \tau_a)(1 - \tau_f)[f'(\hat{k}) - \delta] - x - n]dv)\} = 0$$

  Therefore, in the steady state, where $\hat{k} = \hat{k}^*$, the net marginal product of capital, $f(\hat{k}) - \delta$, must exceed $(x + n)/[(1 - \tau_a)(1 - \tau_f)]$

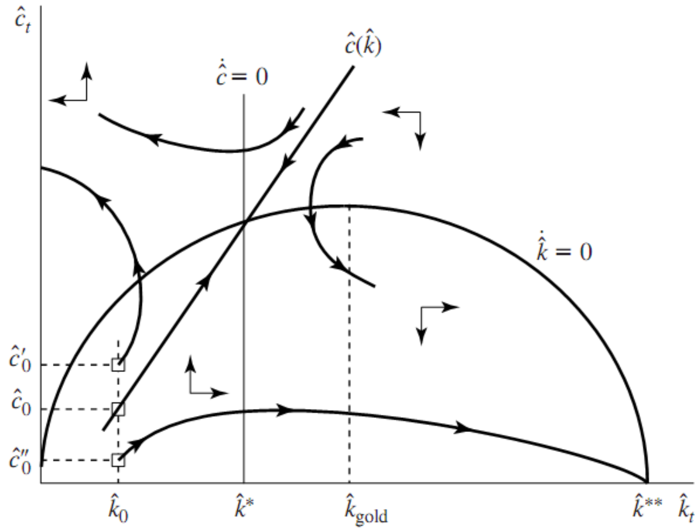
Taxes onWages and Consumption

- The tax rate on wage income, τ_w, does not enter into any of the equilibrium conditions as we assumed that households worked a fixed amount. In this case, a wage tax amounts to a lump-sum, nondistorting tax
- The consumption tax rate, τ_c, in principle, can affect
  - The choice of consumption over time
    - But cannot do that in the present setting since τ_c is time invariant—if the tax rate on consumption is expected to increase in the future, individuals would want to consume more now and less in the future, so consumption growth would be reduced)
  - The labor-leisure choice in a model with divisible labor
    - That is a model in which households choice how much labor to supply
- In the present setting τ_c does not affect the equilibrium and works like a lump-sum tax

Phase Diagram

- Remember that in the basic Ramsey model without the government, we obtained the following two equations and the graph

$$\frac{\hat{c}^{\cdot}}{\hat{c}} = \frac{1}{\theta}[f'(\hat{k}) - \delta - \rho - \theta x] \qquad\qquad \hat{k}^{\cdot} = f(\hat{k}) - \hat{c} - (n + \rho + x)\hat{k}$$



- Now we have the following equations

$$\dot{\hat{k}} = f(\hat{k}) - \hat{c} - (x + n + \delta)\hat{k} - \hat{g} \tag{2}$$

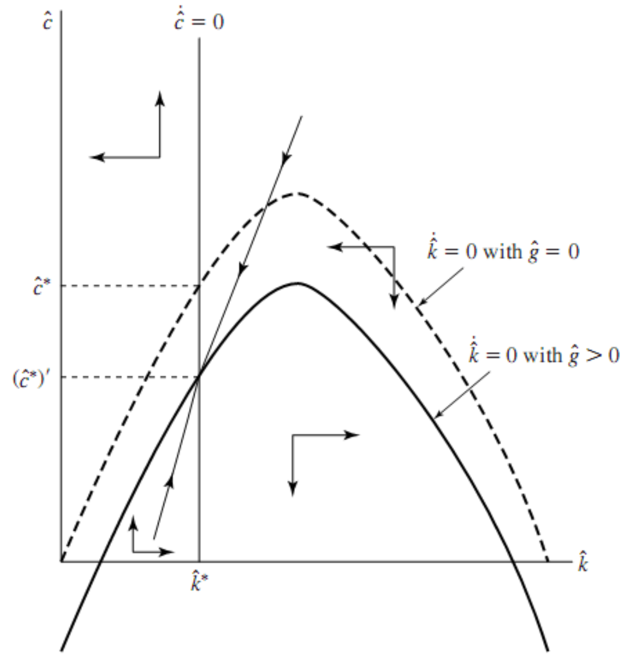
$$\dot{\hat{c}}/\hat{c} = (1/\theta)\{(1 - \tau_a)(1 - \tau_f)[f'(\hat{k}) - \delta] - \rho - \theta x\} \tag{3}$$

- If we assume $\hat{g} = \tau_a = \tau_f = 0$, the phase diagram in $(\hat{k},\ \hat{c})$ space would be exactly as shown in the previous figure

- If we assume instead that $\hat{g} > 0$, then $\dot{\hat{k}} = 0$ locus is displaced downward in accordance with equation (2), so consumption is transfered from households to the useless government expenditure

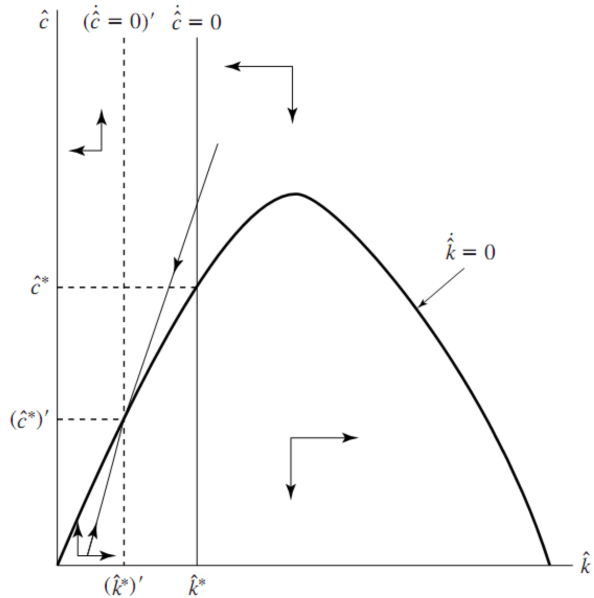
  - Suppose the government purchases, G, would be financed by some combination of τ_w and τ_c. Consider also the time path of transfers, V
  - The precise combination of τ_w, τ_c, and V does not matter because these variables amount to lump-sum taxes or transfers in the model

- The effect of an increase in government purchases



Taxes on Asset Income and Firms' Earnings

- If we assume $\hat{g} > 0$, together with $\tau_a > 0$ or $\tau_f > 0$, equation (3) implies that $\dot{\hat{c}} = 0$ locus shifts to the left without changing the $\dot{\hat{k}} = 0$ locus (though this locus is already below its location in the basic Ramsey model due to the $\hat{g}$)



- As the diagram shows, the imposition of taxes on the income from capital leads to reductions in $\hat{k}^*$ and $\hat{c}^*$ in the long run
- These effects arise because the taxes reduce the incentive to save
- The transversality condition ensures that, after the initial increase in the tax rate at time zero the economy will find itself on the new stable arm
- Since the level of capital cannot jump at time zero, the initial level of consumption has to increase. The reason is that, initially, the increase in taxes motivate people to substitute consumption toward the present instead of saving