D - OVERLAPPING GENERATIONS (OLG) MODEL

1. Households
2. Firms
3. Equilibrium
4. Efficiency
5. Extensions
    - Social Security
    - Government Financing
    - Altruism

Households

- Live two periods
  - Young: Provides 1 unit of labor inelastically and receives the wage income w_t. When s_t denotes the amount saved in period t, the budget constraint for period t is
$$c_{1,t} + s_t = w_t \tag{1}$$
  - Old: Retired, does not work. In period $t+1$, s/he consumes the previous saving plus the accrued interest:
$$c_{2,t+1} = (1 + r_{t+1})s_t \tag{2}$$
- Agents maximize lifetime utility
$$U_t = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\left(\frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta}\right) \quad \text{where} \quad \theta > 0, \ \rho > 0 \tag{3}$$

Intertemporal Budget Constraint (BC)

- We can combine equations (1) and (2) to obtain

$$c_{1,t} + \frac{c_{2,t+1}}{1 + r_{t+1}} = w_t \tag{4}$$

Household's Maximization Problem

$$\max_{\{c_{1,t}, c_{2,t+1}\}} U_t = U(c_{1,t}) + \frac{1}{1+\rho} U(c_{2,t+1}) - \lambda(w_t - c_{1,t} - \frac{c_{2,t+1}}{1 + r_{t+1}})$$

- using FOCs with respect to two choice variables, we obtain

$$\frac{U'(c_{1,t})}{U'(c_{2,t+1})} = \frac{1 + r_{t+1}}{1 + \rho}$$

- Using the isoelastic form of $U(\cdot)$ in (3)

$$\frac{c_{1,t}}{c_{2,t+1}} = (\frac{1+\rho}{1 + r_{t+1}})^{1/\theta}$$

- Inserting the final equation into the Intertemporal Budget Constraint in (4)

$$w_t = [(\frac{1+\rho}{1+r_{t+1}})^{1/\theta} + \frac{1}{1+r_{t+1}}]c_{2,t+1}$$

which finds $c_{2,t+1}$ as

$$c_{2,t+1} = [(1+\rho)^{1/\theta}(1+r_{t+1})^{-1/\theta} + (1+r_{t+1})^{-1}]^{-1}w_t$$

- Combining with equation (2) results in

$$s_t = \frac{1}{1+r_{t+1}} \frac{1}{[(1+\rho)^{1/\theta}(1+r_{t+1})^{-1/\theta} + (1+r_{t+1})^{-1}]} w_t$$

which can be reduced to

$$s_t = [(1+\rho)^{1/\theta}(1+r_{t+1})^{(\theta-1)/\theta} + 1]^{-1}w_t \tag{5}$$

Firms

- Remember from the Solow model that (when $f(k_t)$ is used)

$$r_t = f'(k_t) - \delta \qquad w_t = f(k_t) - k_t f'(k_t)$$

Equilibrium

- The resource constraint in a closed economy is

$$K_{t+1} - K_t = F(K_t, L_t) - C_t - \delta K_t$$

- (or) Net investment = Total factor income - Total consumption

$$K_{t+1} - K_t = w_t L_t + r_t K_t - c_{1,t} L_t - c_{2,t} L_{t-1} \tag{6}$$

- Budget constraints in (1) and (2) can eliminate $c_{1,t}$ and $c_{2,t}$ (*notice that these consumptions are made by different generations living in the same period)

$$K_{t+1} - K_t = w_t L_t + r_t K_t - (w_t - s_t) L_t - ((1 + r_t) s_{t-1}) L_{t-1}$$

this equation further reduces to

$$K_{t+1} = s_t L_t + (1 + r_t)(K_t - s_{t-1}L_{t-1}) \tag{7}$$

- Since K_1 is owned by the L_0 old people living at time t, and since there is no bequest motive, these people consumes everything they have, that is $C_{2,1} = (1 + r_1)K_1$. This condition, together with (1) and (6) implies that

$$K_2 = s_1 L_1$$

Thus (7) implies that

$$K_{t+1} = s_t L_t \quad \text{for } t \geq 2 \tag{8}$$

- That is, *the savings of the young equal the next period's capital stock.*

- Assuming a constant rate of population growth, so that $L_{t+1}/L_t = 1 + n$, the equation (8) can be expressed as

$$\Rightarrow \quad k_{t+1} \equiv K_{t+1}/L_{t+1} = s_t/(1 + n) \quad \text{for } t \geq 1$$

- Using s_t from equation (5)

$$k_{t+1} = \frac{1}{1+n}[(1+\rho)^{1/\theta}(1+r_{t+1})^{(\theta-1)/\theta}+1]^{-1}w_t \tag{9}$$

  inserting the prices

$$k_{t+1} = \frac{1}{1+n}\{1+(1+\rho)^{1/\theta}(1+f'(k_t)-\delta)^{(\theta-1)/\theta}\}^{-1}[f(k_t)-k_t f'(k_t)]$$

  For every value of k_t, this equation determines the equilibrium value of k_{t+1}

Steady State

- At the steady-state capital intensity, $k_{t+1} = k_t = k^*$

$$k^* = \frac{1}{1+n}\{1+(1+\rho)^{1/\theta}(1+f'(k^*)-\delta)^{(\theta-1)/\theta}\}^{-1}[f(k^*)-k_t f'(k^*)]$$

- Now suppose the utility function is logarithmic, $\theta = 1$, and the production function is Cobb-Douglas, $f(k_t) = Ak_t^{\alpha}$. One can show that

$$k^* = [\frac{A(1-\alpha)}{(1+n)(2+\rho)}]^{1/(1-\alpha)}$$

The Golden Rule and Dynamic Efficiency

- Remember that oversaving can arise in the Solow model as it assumes an arbitrary saving rate, but not in the Ramsey model, in which infinite-lived households choose saving optimally

- In the OLG model, oversaving can occur even though households choose saving optimally. This possibility exists because households have a finite horizon, whereas the economy goes on forever

- To see this possibility, first notice that because propulation growth rate is constant, maximization of consumption per capita is equivalent to maximization of consumption per worker.

- Dividing the resource constraint in a closed economy, that is

$$K_{t+1} - K_t = F(K_t, L_t) - C_t - \delta K_t$$

  by L_t, we obtain

$$k_{t+1}(1+n) - k_t = f(k_t) - c_t - \delta k_t$$

- At the steady-state $k_{t+1} = k_t = k^*$; hence,

$$c^* = f(k^*) - (n+\delta)k^*$$

- The golden rule level of consumption occurs where $f'(k_g^*) = (n+\delta)$. The same solution with the Solow model.

- To see the economy's steady-state value k^* may end up in the dynamically inefficient region where the steady-state capital intensity to exceed the golden-rule value , $k^* > k_g^*$, consider the log-utility case, $\theta = 1$, and the Cobb-Douglas technology. Then the condition for the steady-state capital intensity to exceed the golden-rule value is therefore

$$\frac{1-\alpha}{(1+n)(2+\rho)} > \frac{\alpha}{n+\delta}$$

- Thus oversaving is more likely to occur if the rates of time preference, ρ, and population growth, n, are small; if the depreciation rate, δ, is large; and if the capital share, α, is small.

- Oversaving cannot occur if α is close to 1 (because wages are then close to 0, and young people have little capacity to save).
- Notice that in this model every period is about 30, 40 years. So you cannot take n=0.005 as you do with the one year, same for δ. So adjust for the length of period!

Model Dynamics

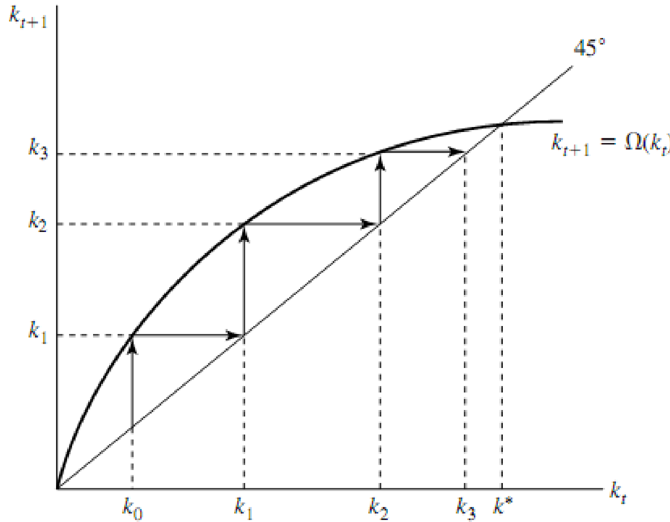
- Remember the equation (9)

$$k_{t+1} = \frac{1}{1+n}[(1+\rho)^{1/\theta}(1+r_{t+1})^{(\theta-1)/\theta}+1]^{-1}w_t \tag{10}$$

- When the utility function is logarithmic, $\theta = 1$, and the production function is Cobb-Douglas, it simplifies to

$$k_{t+1} = (1-\alpha)\cdot Ak_t^{\alpha}/[(1+n)\cdot(2+\rho)]$$

– When k_t goes to 0, the slope of the RHS goes to infinity, when k_t goes to infinity, its slope goes to zero. Therefore



- Equilibrium dynamics are identical to those of the basic Solow model and monotonically converge to k^*

- Sameulson (1958) and Diamond (1965) shaped the class of OLG models we use today
- It contains agents who are born at different dates and have finite lifetimes, even though the economy goes on forever
- This induces a natural heterogeneity across individuals at a point in time, as well as nontrivial life-cycle considerations for a given individual across time
- The basic model can be extended to include Altruism, Bequests, Pension Systems, and Infinite Horizons
- The model has a role for fiat money; hence, can be used to address a variety of substantive issues in monetary economics
- Many of the applications use log preferences ($\theta = 1$). Income and substitution effects exactly cancel each othe: changes in the interest rate (and thus in the capital-labor ratio of the economy) have no effect on the saving rate

Review of the Other Basic Growth Models

The Solow-Swan Model: Saving is exogenous, technology is exogenous

We solved the social planner problem but also showed that if there are markets for capital and labor we would obtain the same solution. The crucial prediction of the model is that only technological process can ensure sustainable growth. Also we can expect convergence in per capita income among countries only if they have the same parameters, such as population, depreciation and saving rates

Endogenous Models: Saving is exogenous, technology is endogenous

If capital is broadened from physical capital to include human capital, law of diminishing returns may not apply (increasing returns to investment from education and efficiency, and innovation not necessary). With no diminishing return to capital the model predicts sustainable growth. The model does not predict, yet, convergence among countries

The Ramsey Model: Saving is endogenous, technology is exogenous

This is a dynamic general equilibrium model. Consumers maximize their utility, firms maximize their profits, then we find the equilibrium of the economy together with the path of consumption and capital while they approach their steady states. The model predicts that optimizing consumers do not save enough to attain golden rule levels of capital and consumption.

Innovation Economics: The main determinants of growth are knowledge, entrepreneurship, technology and innovation are not independent forces. Their interaction not only results in growth but also requires improved economic policy. Besides, institutions, culture, norms and networks are central to growth. Joseph Schumpeterian is the first who studied on innovation and argued that the innovation and technological change of a nation comes from the entrepreneurs, or wild spirits. This is how he explains the technological progress and approaches to endogenous growth.