TOBB-ETU, Economics Department

Macroeconomics II (ECON 532)

Practice Problems I

Q: The Solow-Swan Model: Constant returns

Prove that, if the production function exhibits constant returns, all firms will use the same proportion of capital to labor.

Q: The Solow-Swan Model: Shares and Golden Rule

a-) Consider a Solow model without a technological growth. Derive the Golden Rule of Capital Accumulation. What conditions (parameters) determine the golden rule capital stock?

b-) Consider a Cobb Douglas technology with parameter $0 < \alpha < 1$. Imagine that the depreciation rate of capital is $\delta = 0.10$ and the rate of population growth is $n = 0.01$. Compute the golden rule capital stock k_{gold} as a function of α. What is the golden rule capital stock when $\alpha = 1/2$?

c-) Imagine that the saving rate in the economy is the constant "s" (with $0 < s < 1$). What would be the value of s that delivers k_{gold} as the steady state capital stock?

d-) Suppose now that inputs earn their marginal products. Show that when owners of capital save all their income and workers consume all their income, then the economy reaches the golden rule capital stock

Q: The Solow-Swan Model: Harrod-Domar

Consider the Solow-Swan model with constant savings rate, s. Imagine that the technology is Leontief so: $Y = \min(AK, BL) \quad (1)$

b-) Write down $y = Y/L$ as a function of k

c-) If the rate of population growth is zero ($n = 0$), show that the growth rate of capital is as follows:

Draw the savings and the depreciation lines as a function of k. Find the steady state capital stock, k^*, for each of the cases written below. What are the dynamics of k over time? Do you see any problems with those steady states?

d-) $sA > \delta$ **e-)** $sA < \delta$ **f-)** $sA = \delta$

Q: The Solow-Swan Model: CES (Constant Elasticity of Substitution) Production Function

Consider the following production function

$$Y = F(K, L) = A(a[bK]^{\psi} + (1-\alpha)[(1-b)L]^{\psi})^{1/\psi}$$

where $0 < a < 1$, $o < b < 1$ and $\psi < 1$

a-) Show that the elasticity of substitution between capital and labor is $\dfrac{1}{1-\psi}$.

b-) Show that when $\psi \to 0$ (unit elasticity of substitution), the production function approaches the following Cobb Douglas technology

$$Y = cons\tan t \cdot K^{\alpha} L^{1-\alpha}$$

c-) Express output per capita as a function of capital per capita

d-) Compute the marginal product of capital and the average product of capital

e-) Under what conditions of the parameters the Solow-Swan model with a CES production function displays *a steady state with zero growth*, and displays *endogenous growth*? Explain intuitively

Q: The Solow-Swan Model: Conditions for the Existence of Steady State

Suppose the production function is of the form

$$F(K, L) = \frac{KL}{.25L + .5K}$$

Derive the function for the steady state of capital and give conditions under which a steady state will exist. Discuss the possibility of k increasing indefinitely or decreasing indefinitely (to zero).

Q: The Solow-Swan Model: Capital flows in the Solow model

Suppose that there are two countries, Scotland and Canada. Both countries are well-described by the Solow model and have identical Cobb-Douglas production functions:

$$F(K, L) = AK^{\alpha}L^{1-\alpha}$$

The countries have the same level of technology (A) but have different capital to labor ratios, and thus different per capita income. Define the gross return on capital as $r_t + 1 - \delta$. Let $\delta = 0$, and $\alpha = 1/3$.

a-) If Canada has 10 times Scotland's per capita income, what is the ratio of per-capita capital in Canada to that in Scotland?

b-) If Canada has a 5 percent net return on capital, (and 10 times the per capita income of Scotland), what is the net return on capital in Scotland?

c-) If you were a Canadian under these circumstances with some savings to invest, would you invest it in Canada or send it to Scotland?

Q: Ramsey Model: Harrod-Domar

Production function is given as Leontief: $Y = \min(AK, BL)$. Suppose that agents are optimizing. Imagine that the dynamic system characterizing the solution is as usual under the assumption of zero growth rates of population and technology

$$\dot{k} = f(k) - c - \delta k \quad \text{and} \quad \frac{\dot{c}}{c} = \frac{1}{\theta}(f'(k) - \delta - \rho)$$

a-) Draw the locus for $\dot{k} = 0$ on the $c - k$ space (make sure you distinguish two regions: $k \lesseqgtr B/A$)

b-) Draw $f(k)$ as a function of k. Notice that the derivative $f(k)$ is not defined at $k = B/A$ (to the left of $k = B/A$ we have A, and to the right of $k = B/A$ we have 0), but assume that it is $\delta + \rho$. Then draw the locus for $\dot{c} = 0$ on the $c - k$ space

c-) Draw the direction of c and k with arrows on the four regions separated by $\dot{k} = 0$ and $\dot{c} = 0$ locuses

d-) Suppose that the steady state occurs where $\dot{k} = 0$ and $\dot{c} = 0$ locuses intersect with positive level of capital and consumption. Discuss if there is any idle factor in the economy

Q: Ramsey Model

Consider the Ramsey economy with technological growth, where the consumption and capital law of motions are given by

$$\hat{k}^{\cdot} = f(\hat{k}) - \hat{c} - (n + \delta + x)\hat{k} \quad \text{and} \quad \frac{\hat{c}^{\cdot}}{\hat{c}} = \frac{1}{\theta}(f'(\hat{k}) - \delta - \rho - \theta x)$$

with TVC

$$\lim_{t\to\infty} \hat{k} \exp\{-\int_0^t [f'(\hat{k}) - \delta - n - x] dv\} = 0$$

which implies that at the steady state

$$f(\hat{k}^*) - \delta > n + x.$$

Now assume that the model is on the balanced growth path with steady state values of $\hat{k}^*$ and $\hat{c}^*$. What are the effects of a decline in the depreciation rate, δ, on the $\dot{k} = 0$ and $\dot{c} = 0$ locuses, and on the $\hat{k}^*$, $\hat{c}^*$ and $\hat{y}^*$. Explain your results and give a brief economic interpretation.