TOBB-ETU, Economics Department

Macroeconomics II (ECON 532)

Practice Problems II

Q: Ramsey Model: Exponential Utility

Assume that infinite-horizon households maximize a utility function of the exponential form

$$\max U = \int_0^\infty e^{(n-\rho)t} - (1/\theta)e^{-\theta c} dt \qquad \text{where} \;\; \theta > 0$$

The behavior of firms is the same as in the Ramsey model, with zero technological progress.

a) Find the first-order conditions for a representative household with preferences given by this form of u(c).

b-) Relate θ to the concavity of the utility function and to the desire to smooth consumption over time. Compute the intertemporal elasticity of substitution. How does it relate to the level of per capita consumption, c?

c-) Combine the first-order conditions for the representative household with those of firms to describe the behavior of c and k over time. [Assume that $k(0)$ is below its steady-state value.]

d-) Derive the growth rates for the decentralized economy and for the social planner. Comment on differences between this economy and the market solution.

Q: Ramsey Model: Spillovers from Aggregate Capital Stock

Consider a ramsey model where the utility function of consumers has the CRRA form

$$\max U = \int_0^\infty e^{-\rho t} \frac{c^{1-\theta} - 1}{1-\theta} dt$$

Suppose the firm i's productivity depends on the economy's on the aggregate capital stock, K

$$Y_i = AK_i^\alpha L_i^{1-\alpha} K^{1-\alpha}$$

Derive the growth rates for the decentralized economy and for the social planner. Comment on differences between these two solutions. Finally discuss if the scale effect appears in the solutions

Q: Ramsey Model: Optimal Growth - The Cake-Eating Problem (*You are not responsible from this example)

Consider the optimal growth problem (discrete time) where: $f(k) = k$. This problem is commonly called a "cake-eating" problem. The consumer starts with a certain amount of capital, and "eats" it over time. We will use this problem to try some dynamic programming. The planner's problem is to maximize:

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

subject to the constraints:

$$\begin{aligned} k_{t+1} &\leq k_t - c_t \\ k_t &\geq 0 \\ k_0 &> 0 \end{aligned}$$

where log is the natural (base e) logarithm function.

a-) Write down Bellman's equation for this problem.

b-) First we perform policy function iteration. We start with guessing the optimal policy (control variable as a function of state variable) as

$$c_0(k) = (1-\beta)k$$

Write down the value of k_t (for any $t > 0$) as a function of k_0, β, and t if this policy is followed.

c-) Write down the value of c_t as a function of k_0, β, and t if this policy is followed.

d-) What is the value function if the equation above describes the optimal policy? (to keep things simple, feel free to drop any constant term).

e-) The next step is to calculate the optimal policy under the new value function. This will give you a new policy function, which we will call $c_1(k)$. Find $c_1(k)$.

f-) To find the true optimal policy, you keep applying these two steps until your policy function stops changing i.e., $c_i(k) = c_{i+1}(k)$. What is the true optimal policy function?

g-) Next we will try value function iteration. First we guess at the form of the value function. Suppose that your initial guess for the value function is:

$$V_0(k) = \log k$$

Next, we calculate a new value function according to the formula:

$$V_{i+1}(k) = \max_{c\in[0,k]} \{\log c + \beta V_i(k-c)\}$$

Calculate V_1, V_2, and V_3. (Feel free to throw out any constant terms.)

h-) If you've done it right, you should see a pattern. Use this pattern to discern V_i for an arbitrary integer i.

ı-) What is the limit of this as $i \to \infty$?

i-) What is the optimal policy function when this is the value function?

j-) Try to solve the same problem the way we used in the lecture notes

Q: Ramsey Model: Optimal growth with Logarithmic utility

Consider an optimal growth model with 100% depreciation every period. The social planner maximizes:

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

subject to the constraints:

$$\begin{aligned} k_{t+1} &= Ak_t^{\alpha} - c_t \\ k_t &\geq 0 \\ k_0 &> 0 \end{aligned}$$

a-) Write down the Bellman equation for this planner's problem.

b-) Write down the necessary conditions for a solution to this planner's problem

c-) Derive the Euler equation.

d-) Find the steady state values of k_t, c_t, and y_t.

e-) Find the steady state savings rate.

f-) Suppose that $\alpha = 1$, so that we have an "AK" model. Ignore for the time being the possibility that there is no solution and assume that the Euler equation applies. What is the growth rate of consumption?

Q: Exercise on Income Taxation in the Ramsey Model

Consider a ramsey model where the utility function of consumers has the CRRA form

$$\max U = \int_0^{\infty} e^{-(\rho-n)t} \frac{c^{1-\theta} - 1}{1-\theta} dt$$

Government taxes family's incomes and the revenue collected will be distributed to households in the form of lump-sum transfers

$$\dot{a} = (1-\tau)(\omega + ra) - c - na + z$$

Derive the growth rates for the decentralized economy and for the social planner. Comment on differences between these two solutions.

Q: Ramsey Model: Congestion of Public Services (G/Y)

Consider a ramsey model where the utility function of consumers has the CRRA form

$$\max U = \int_0^\infty e^{-(\rho-n)t}\frac{c^{1-\theta}-1}{1-\theta}dt$$

Suppose that output for firm i is given by

$$Y_i = AK_i(g/y)^\alpha$$

and for single firm

$$y_i = Ak_i(g/y)^\alpha$$

that is governmental activities (highways, water systems, police and fire services, and courts) serve as an input to private production. G is financed by a proportional tax on households' earnings

$$\dot{a} = (1-\tau)(\omega + ra) - c - na$$

This final equation also states that $g/y = \tau$ and τ is fixed. Derive the growth rates for the decentralized economy and for the social planner. (Assume that the social planner takes g/y given as well.) Comment on differences between these two solutions

Q: OLG Model: Dynamic inefficiency

Remember the OLG model we solved in the class with a Cobb-Douglas production function and log-utility. Suppose population growth rate, n, is 0 and depreciation rate, δ, is 1.

a-) Solve for the worker's savings rate.

b-) Solve for the golden rule level of the capital stock.

c-) Solve for the steady-state savings rate needed to maintain the golden rule level of the capital stock.

d-) Under what conditions on the model parameters will the savings rate exceed the golden rule level?

e-) Prove that these conditions imply that the equilibrium is inefficient by finding a Pareto superior allocation.

f-) Put this result in words. I suggest something of the form "Dynamic inefficiency is more likely if capital's share is (low?/high?) and if young people care (more?/less?) about their future consumption."

g-) Will this model exhibit multiple equilibria? How do you know this?

h-) Will this model exhibit history-dependence? How do you know this?

Q: OLG Model: Social Security using Taxes

Consider an OLG model where rete of technological progress is zero, production is Cobb-Douglas, population grows at a rate n, and utility is logarithmic in a way that

$$\sum_{t=0}^{\infty}(\frac{1}{1+\rho})^t \log c_t$$

Suppose the government taxes each young individual an amount T and uses the profits to pay benefits to old individuals; thus each old person receives $(1+n)T$

a-) Write down the budget constraint faced by an individual born at time t.

b-) Use the budget constraint to determine the saving function of this individual.

c-) How, if at all, does the introduction of the pay as you go social security affect the relation between k_t and k_{t+1} as we found in the clas [Hint: Use the individual savings function obtained in part (b) and proceed as usual to determine the relationship between k_t and k_{t+1}.]

d-) How, if at all, does the introduction of this social security system affect the balanced growth path value of k?

e-) If the economy is initially on a balanced growth path (BGP) that is dynamically efficient, how does a marginal increase in T affect the welfare of current and future generations? What happens if the initial BGP is dynamically inefficient?