TOBB-ETU, Economics Department
Macroeconomics II (ECON 532)
Practice Problems III

Q: Consumption Theory (CARA utility)

Consider an individual living for two periods, with preferences

$$U(c_1, c_2) = u(c_1) + \frac{1}{1+\rho} u(c_2), \text{ where } u(c) = -\frac{1}{\gamma} \exp(-\gamma c) \tag{1}$$

The individual receives a deterministic income $\{y_1, y_2\}$ in the two periods. Saving/borrowing can be made at the interest rate r.

a) Write down the dynamic maximization problem of the individual as a Lagrangian and derive the Euler equation.

b) Using the approximation $ln(1+x) \simeq x$, solve for the optimal consumption choices $\{c_1^*, c_2^*\}$ as a function of $(\gamma, r, \rho, y_1, y_2)$. Under which condition is c_2^* larger than c_1^*?

Q: Linear Utility

Consider the intertemporal consumption-savings problem of an infinitely-lived consumer with assets A_0 at the beginning of period zero This consumer receives a fixed return R on her assets that is paid out at the end of the period. He also has a stochastic labor income y_t that is realized before the consumption choice in every period, subject to the usual No-Ponzi GC and a non-negativity constraint on consumption, $c_t \geqslant 0$ for all t. Suppose the consumer has linear preferences over consumption, $u(c) = \gamma c$. What is the optimal path for consumption if $\beta R < 1$, $\beta R > 1$ and $\beta R = 1$? Explain.

Q: Quadratic Utility and Fixed Income

Let's assume that $\rho = r$, and consumers preferences are represented by a quadratic utility function $u(c) = c - b/2 \cdot c^2$. When we used these two assumptions together with the following Euler Equation

$$E_t u'(c_{t+1}) = \frac{1+\rho}{1+r} u'(c_t)$$

we found that $E_t c_{t+1} = c_t$. Using this equality, and assuming that $y_t = \bar{y}$,

a-) Find consumption at time t in terms of wealth and income of the consumers, $c_t(A_t, \bar{y})$, by using the following Present Value Budget Constraint

$$\sum_{i=0}^{\infty} (\frac{1}{1+r})^i E_t c_{t+i} = \sum_{i=0}^{\infty} (\frac{1}{1+r})^i E_t y_{t+i} + (1+r) A_t$$

and interpret your finding

b-) Find the transitory income y_t^T, and the relation between A_t and A_{t+1} by using the Budget Constraint $A_{t+1} = (1+r)A_t + y_t - c_t$, and interpret your findings

Q: Dynamic Programming: Quadratic Utility and no Labor Income

Remember the Bellman Equation

$$V_t[(1+r)A_t + y_t] = \max_{c_t}\{u(c_t) + \frac{1}{1+\rho}E_t V_{t+1}[(1+r)A_{t+1} + y_{t+1}]\}$$

which is subject to

$$A_{t+1} = (1+r)A_t + y_t - c_t$$

Now suppose that there is no labor income and all income is derived from tradable wealth. Further suppose that $r = \rho$ and the utility of the consumer is of the functional form: $u(c) = c_t - b/2 \cdot c_t^2$. Use the Bellman equation to solve consumption, c, in terms of A

- Some additional Information:
  - All CRRA, CARA, and quadratic utility functions are the class of HRRA (Hyperbolic absolute risk aversion) utility function
  - Merton shows that the value function is of the same functional form as the utility function for the HRRA utility functions if labor income is fully diversified, or where there is no labor income at all and all income is derived from tradable wealth

Q: Quadratic Utility and Expected Permanent Change in Income

Let's assume that $\rho = r$, and consumer preferences are represented by a quadratic utility function $u(c) = c - b/2 \cdot c^2$. When we use these two assumptions with the Euler Equation:

$$E_t u'(c_{t+1}) = \frac{1+\rho}{1+r}u'(c_t)$$

we found that $E_t c_{t+1} = c_t$. Using this equality, and also assuming that

$$E_t(y_{t+i}) = \begin{cases} \bar{y} & i = 0 \\ 2\bar{y} & i = 1, 2, ...\infty \end{cases}$$

a-) Find consumption at time t in terms of wealth and income of the consumers, $c_t(A_t, \bar{y})$, by using the following Present Value Budget Constraint

$$\sum_{i=0}^{\infty}(\frac{1}{1+r})^i E_t c_{t+i} = \sum_{i=0}^{\infty}(\frac{1}{1+r})^i E_t y_{t+i} + (1+r)A_t$$

and interpret your finding

b-) Find the transitory income y_t^T, and the relation between A_t and A_{t+1} under this case by using the Budget Constraint $A_{t+1} = (1+r)A_t + y_t - c_t$ and interpret your findings

Q: Quadratic Utility Function and Unexpected Permanent Change in Income

Let's assume that everything is the same with previous question, except that the consumer does not know at time t whether her/his income will change to $2\bar{y}$. So

$$E_t(y_{t+i}) = \bar{y} \quad for \;\; i = 1, 2, ...\infty$$

but the reality is

$$y_{t+i} = \begin{cases} \bar{y} & i = 0 \\ 2\bar{y} & i = 1, 2, ...\infty \end{cases}$$

and finally, once his income is increased to $2\bar{y}$ at time $t+1$, he changes his expectations as

$$E_{t+1}(y_{t+i}) = 2\bar{y} \qquad i = 2, ...\infty$$

a-) Find the difference between consumptions at time $t+1$ and at time t in terms of wealth and income of the consumers, $c_{t+1}(A_{t+1}, \bar{y}) - c_t(A_t, \bar{y})$, by using the following Present Value Budget Constraint (Hint: you need to solve this constraint both at time t and $t+1$)

$$\sum_{i=0}^{\infty}(\frac{1}{1+r})^i E_t c_{t+i} = \sum_{i=0}^{\infty}(\frac{1}{1+r})^i E_t y_{t+i} + (1+r)A_t$$

and interpret your finding

b-) Find the transitory income y_t^T, and the relation between A_t and A_{t+1} by using the Budget Constraint at time t, $A_{t+1} = (1+r)A_t + y_t - c_t)$, and interpret your findings

Q: Dynamic Programming with Log Utility

Remember the Bellman Equation

$$V_t[(1+r)A_t + y_t] = \max_{c_t}\{u(c_t) + \frac{1}{1+\rho}E_t V_{t+1}[(1+r)A_{t+1} + y_{t+1}]\}$$

which is subject to

$$A_{t+1} = (1+r)A_t + y_t - c_t$$

Suppose that there is no labor income and all income is derived from tradable wealth. And suppose that the utility of the consumer is of the functional form: $u(c) = \log c$. Use the Bellman equation to solve consumption, c, in terms of A

Q: Dynamic Programming: Consumption Theory with CRRA utility

Consider an individual with an infinite horizon and CRRA preferences

$$\sum_{j=0}^{\infty} \beta^j \frac{c_{t+j}^{1-\gamma}}{1-\gamma} \tag{1}$$

where $\gamma \geq 1$ is the inverse of the Intertemporal elasticity of substitution. The individual faces a known deterministic income stream$\{y_{t+j}\}_{j=0}^{\infty}$ and saves/borrows at the interest rate r.

a) Write down the Bellman Equation and derive the Euler equation.

b) What is the optimal path of consumption when $\beta(1+r) = 1$?

Q: Proportional Earnings Tax

Consider the permanent income hypothesis in the deterministic case with quadratic utility and $\beta(1+r) = 1$. Suppose the government, unexpectedly, at date t introduces a proportional tax τ on earnings, meaning that it taxes away a fraction τ of earnings.

a) Imagine this tax is transitory, meaning that it is effective only at date t. Determine by how much consumption falls compared to the no tax case.

b) Imagine this tax is permanent, meaning that it is effective at every date $t+j$ with $j \geqslant 0$. Determine by how much consumption falls compared to the no tax case.

Q: Certainty Equivalence

Consider the standard consumption problem with stochastic labor income y_t. The consumer's maximization problem is

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

given an initial level of assets A_0 and subject to the usual no-Ponzi game condition and the dynamic budget constraint

$$A_{t+1} = RA_t + y_t - c_t$$

where $R = 1+r$ is the (risk-free and constant) return on asset holdings. Suppose the utility function is quadratic.

$$u(c) = -1/2\theta(\gamma - c)^2, \quad \theta > 0$$

a) Suppose $y_t = y$. Write down the Bellman equation and derive the Euler equation for consumption.

b) Assume that $\beta R = 1$. What is the optimal policy for consumption (c as a function of the state variable A)? [Hint: Use the method of undetermined coefficients: guess a functional form for the policy rule, verify that it satisfies the differential equation for all values of the state variable and calculate its coefficients].

c) Show that the condition $\beta R = 1$ implies that consumption is constant over time. Is there any other reason in this model why consumption might be constant over time?

d) Now let y_t be an i.i.d. random variable. What is/are the state variable(s)? Why?

e) Write down the Bellman equation and derive the stochastic Euler equation for consumption. Again assume $\beta R = 1$. What is the optimal policy in this case?

f) In what sense is the stochastic model equivalent to the deterministic model and in what sense is it not?

Q: Doctorate: Stochastic Asset Returns

Consider the intertemporal consumption allocation decision of a consumer who does not earn any labor income but lives of the return R she gets on her assets A. The dynamic budget constraint is given by

$$A_{t+1} = R_{t+1}(A_t - c_t)$$

Asset returns R_t are an i.i.d. random variable. Because the consumer cannot perfectly forecast the returns on her assets in the future, she solves a stochastic optimization problem, maximizing the expected net present value of utility over consumption

$$\sum_{t=0}^{\infty} \beta^t E_t\left(\frac{c^{1-\theta}}{1-\theta}\right) \tag{1}$$

a) What are the coefficient of relative risk aversion and the coefficient of absolute risk aversion for these preferences? What does that mean?

b) Write down the Bellman equation and derive the Euler equation for consumption.

c) Solve for the policy rule.

Q: Doctorate: Log-normal Approximation

In the model of the previous question, suppose that the interest is constant. Suppose also that income is i.i.d. with some (unspecified) distribution that guarantees that the distribution of c_{t+1} conditional on information at time t is log-normal. Further, assume that the conditional variance of $\log c_{t+1}$ is constant over time and equal to σ^2. The consumer has a CRRA equal to θ.

a) Show that marginal utility is log-normal as well. What are the mean and variance of $\log u'(c_{t+1})$ conditional on information at time t?

b) Show that $\log c_t$ follows a random walk (with drift). [Hint: Substitute the functional form for the utility function into the Euler equation and take logaritms on both sides. Evaluate the expectation on the right-hand side using the fact that c_{t+1} is log-normal and $Var_t[\log c_{t+1}] = \sigma^2$]

c) How do changes in interest rates and the volatility of consumption affect consumption growth. Explain the economic intuition for those results.

Q: Doctorate: Different Stochastic Processes for Income

Consider the consumption-savings choice of an agent whose dynamic budget constraint is given by

$$A_{t+1} = R(A_t + y_t - c_t)$$

Asset returns R are fixed, but labor income y_t is stochastic.

1. Suppose income follows an AR(1) process

   $$y_t = \rho y_{t-1} + \varepsilon_t$$

   where ε_t is i.i.d. and has mean zero and $\rho < 1$. What is the Euler equation? What variables does the policy rule for consumption depend on?

2. Now suppose income follows an AR(2) process

   $$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \varepsilon_t$$

   where $\rho_1 + \rho_2 < 1$. What is the Euler equation? What variables does the policy rule for consumption depend on?

3. Now suppose income follows an MA(1) process

   $$y_t = \varepsilon_t + \phi \varepsilon_{t-1}$$

   where we assume the consumer observes y_t but not ε_t. What is the Euler equation? What variables does the policy rule for consumption depend on?

Q: The Search and Matching Model (p)

Recall the three equations (Beveridge Curve, Job Creation Condition and Wage Condition) for the equilibrium conditions of the Search and Matching Model

$$BC: u = \frac{\lambda}{\lambda + \theta q(\theta)} \qquad JC: p - w = \frac{(r+\lambda)pc}{q(\theta)} \qquad WC: w = (1-\beta)z + \beta p(1 + c\theta)$$

Analyze the effect of an increase in productivity (from p' to p'') graphically on w, u, v. Also give an economic interpretation when you move the graphs

Q: The Search and Matching Model (λ)

Recall the three equations (Beveridge Curve, Job Creation Condition and Wage Condition) for the equilibrium conditions of the Search and Matching Model

$$BC : u = \frac{\lambda}{\lambda + \theta q(\theta)} \qquad JC : p - w = \frac{(r+\lambda)pc}{q(\theta)} \qquad WC : w = (1-\beta)z + \beta p(1 + c\theta)$$

Analyze the effect of an increase in the job separation rate (λ) graphically on w, u, v. Also given an economic interpretation when you move the line and curves

Q: Eliminating unemployment

"Eliminating unemployment is a simple matter of reducing workers' bargaining power". Discuss this claim using the Search-Matching model. Then analyze the outcome of the model when firms have no bargaining power. Contrast the two cases.

Q: SM-Minimum Wages

Consider a standard Search-Matching model but all jobs have the same productivity P and firms post vacancies at a flow cost equal to C. (Remember that in the class we assumed that firms post vacancies at a flow cost equal to pc.) The positive unemployment income of workers is equal to z.

a) Derive the Job Creation and the Wage Curves.

b) The government imposes a minimum wage w_m on firms. Obviously, if w_m is too low it has no effect and if it is too high, firms do not open vacancies. Characterize these two bounds $[\underline{w}, \bar{w}]$
What is the impact of the minimum wage on θ and u?

c) Consider once again that the cost of posting a vacancy is proportional to P, so that $C = cP$ We also assume that the flow income of workers is proportional to P, so that $Z = zP$. What is the new expression of the equilibrium wage?

d) What is the impact of a change in P on wages, θ and unemployment ?

e) Assume that there is technological progress so that $P(t)$ is an increasing function of time. What happens as t goes to infinity if the minimum wage grows at a slower rate than $P(t)$? What happens as t goes to infinity if the minimum wage grows at a faster rate than $P(t)$?