4- Current Method of Explaining Business Cycles: DSGE Models

Basic Economic Models

- In Economics, we use theoretical models to explain the economic processes in the real world. These models define a relation between sets of variables. In this chapter we will work over Dynamic Stochastic General Equilibrium (DSGE) models, but first we look at a couple of economic models and economic terms

Stochastic version of Samuelson's (1939) Classical Model (An Early Macro Model)

$$y_t = c_t + i_t \tag{1}$$

$$c_t = \alpha y_{t-1} + \epsilon_{ct} \tag{2}$$

$$i_t = \gamma(c_t - c_{t-1}) + \epsilon_{it} \tag{3}$$

- Is this a model or system of equations?
  - Both. It is a model with three equations.
- Is this a dynamic or static model?
  - A dynamic one (Some past variables affect the current variables. In the static model everything occurs within the same period)

- What are ϵ's above?
  - They are disturbances (shocks). These are necessary because we cannot expect the real data of variables to move one-to-one in accordance with the relations put by these equations. The unexpected movements in the variables, or put it differently, the changes in the variables that cannot be modelled, goes to the disturbances. As these disturbances are random, they make the model a stochastic one
    - * Putting these terms together, this is a Dynamic Stochastic Model

- Is this a structural model or a reduced form one?
  - It is structural because it explains endogenous variables (that are on the left hand side, model attempts to explain) with current realizations of other endogenous variables. A reduced form model explains endogenous variables with their and other endogenous variables' lags, also current and past values of exogenous variables. For ex., reduce form equation for investment can be obtained as

    $i_t = \gamma(c_t - c_{t-1}) + \epsilon_{it} = \gamma(\alpha y_{t-1} + \epsilon_{ct} - c_{t-1}) + \epsilon_{it} = \gamma\alpha y_{t-1} - \gamma c_{t-1} + \gamma\epsilon_{ct} + \epsilon_{it}$

    if you attempt to estimate this equation, by OLS for example,

    $$i_t = \beta_1 y_{t-1} + \beta_2 c_{t-1} + e_t$$

    you estimate for reduced form parameters and disturbance (β_1, β_2, e_t). Though there are also estimation methods that you can use to estimate directly the structural form parameters

- How can we understand if it is a good or useless model?
  - There can be different models that use the same exogenous variables to explain endogenous variables. A good theoretical model is the one that is successful in doing that. Put it differently, a model is a good one that makes a good fit to the data (the goodness of fit). There are different methods of estimating parameters of a model and measuring its goodness of fit. For each estimation method, we define an objective function, and try to find a set of model parameters that minimizes that. For example in the regression analysis, the objective function is minimizing sum of squared residuals

- Is it a (partial or general) equilibrium model?
  - This is not an equilibrium framework. It doesn't explain the underlying fundamental relation between the variables by aggregating the behavior of individuals and firms. i.e. it does not explain the behavior of supply, demand and prices in a whole economy with several markets by seeking to prove that equilibrium prices for goods exist. It just expresses a theoretical link between aggregate variables, which Lucas criticized
  - The next section gives two examples of Dynamic Stochastic Model with General Equilibrium framework

5- Some DSGE Models: RBC Models

- The utility of a representative consumer

$$\max E[\sum_{t=1}^{\infty} \beta^t U(C, 1 - N_t)] \qquad \beta < 1$$

where N is the amount of labor employed in the production by the consumer. S/he is endowed with 1 unit of labor and $1 - N$ is the leisure. The leisure in the utility function means that s/he dislikes working and gets utility from leisure as well as consumption

This equation is subject to the resource constraint

$$C_t + I_t = Y_t$$

dynamic equation for capital (capital accumulation)

$$K_t = I_t + (1 - \delta) K_{t-1}$$

the (neoclassical) production function

$$Y_t = Z_t F(K_{t-1}, A_t N_t)$$

exogenous processes for technology

$$A_t = \gamma A_{t-1} \quad \gamma > 1$$

$$\log Z_t = (1-\psi)\log \bar{Z} + \psi \log Z_{t-1} + \epsilon_t \qquad \psi < 1 \qquad \epsilon_t \sim N(0, \sigma^2)$$

- This problem is different than the Ramsey Economy in two respects
  - There is Z_t in the production function, which governs short term movements in the technology, whilst A_t stands for the growth component of technology
  - The leisure (or labor) affects utility

- When this is the case, agents optimize their labor decision in response to technological shocks. Thus, the model is able to explain fluctuations in the employment that we observe in the real economies

- This is the problem of benevolent social planner (like in the ideal socialist economy). S/he tries to maximize the utility of citizens given the resource constraints in the economy. We could have written the same problem by defining consumers trying to maximize their utility and firms trying to maximize their profits (just like in a capitalist economy with free markets). As long as there is a perfect competition on the side of firms, competitive markets give the same solution with the social planner

- Referring to the (neo-classical) production function

$$Y_t = Z_t F(K_{t-1}, A_t N_t)$$

exogenous process for technologies

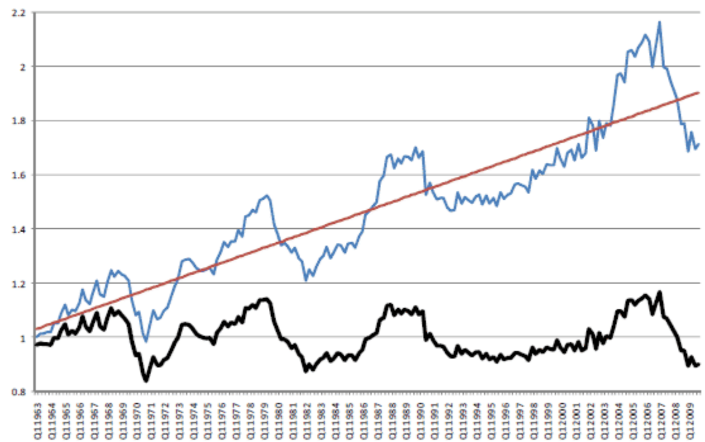
$$A_t = \gamma A_{t-1} \quad \gamma > 1 \qquad \text{and} \qquad \log Z_t = (1-\psi)\log \bar{Z} + \psi \log Z_{t-1} + \epsilon_t$$

  – Timing: Realize that current output (Y_t) depends on the amount of capital that is decided previous period (K_{t-1}). This is just a dating convention and not an obligation. Since in the real economy it takes some time to make an investment and increase the stock of capital, in this model the current output is the output that can be produced with the capital in hand

– Technology: A_t is the technological process used to explain economic growth. This technology grows at a constant rate (γ) at each period, so does the economy

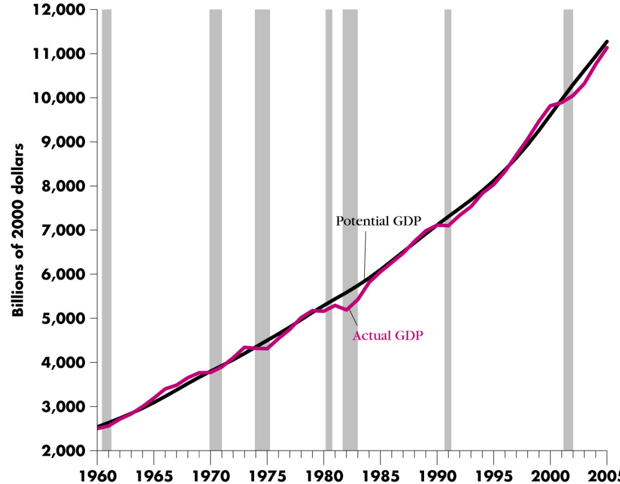
  * However, we know from the data that GDP does not grow at a constant rate, but show some fluctuations. That is why we need Z_t in the model. It is technological process as well but it is only used to explain short run deviations from the trend of growth. It is the heart of RBC theory, as it assumes that fluctuations in the economy comes from the supply (production) side and technology affects production directly
  * Z_t has a steady state value ($\bar{Z}$), which is subject to shocks (ϵ_t). The shocks first affect Z_t, but their effect lasts more than one period (because Z_{t+1} is affected by Z_t, also Z_{t+2} is affected by Z_{t+1}, and so on)

- Suppose in the figure below the blue line (volatile and increasing) is the Real GDP. We can think of the data in two components. Growth component (red line: increasing and straight) and the volatile component (black line: volatile and not increasing). In our notation we explain the red line with A, and the black with Z



- – Instead of estimating the rate of technological growth (γ) and the parameters of volatile component of of technology (ψ, σ) at the same time by using the blue line, we usually can estimate them separately after decomposing the data into red and black ones
  – Since we are interested in the volatile component of the data (fluctuations), I will not use the growing component of technology, A, and use $Y = ZF(K, L)$
  – The deviation of output from its long term trend can be found by *detrending* the data. However, in reality GDP does not always

grow at a constant rate (as it is seen below by the volatile red line)



– There are detrending methods that not only takes away the linear trend, but also finds the fitted black line, so that we can obtain better measure of cyclical behavior of the data (deviation of data from long term growth rate)

– For our coming purposes detrending is enough. But for some other purposes it is not. To see this, notice that detrending collects all the irregular movements in the data left over after throwing away the growth component. These irregular movements (cycles) include cycles of high frequency (having short period) and the cycles of shorter frequency. However, in the Economics, cycles of 6 to 24-32 quarters is considered of crucial importance. So to extract Business Cycles of only this length, we use *filtering* methods (filtering the shorter frequency cycles also mitigates the measurement errors). Hence, filtering methods are able to do the same job with the detrending methods, but also they are able to filter the data so that we would have only the Business Cycles at the interest of Economics. Moreover, filtering methods date turning points of the Business Cycles

- How to solve a RBC model?
  - We can reduce equations into two

$$\max E[\sum_{t=1}^{\infty} \beta^t U(C, 1 - N_t)]$$

and

$$Y_t + (1 - \delta)K_{t-1} = C_t + K_t$$

The left hand side shows the total amount of resources at the beginning of time t. These resources can be spent either on consumption, C_t, or to purchase the capital stock at time t

- Now write it in Lagrangian form and take FOCs w.r.t. C_t, N_t, K_t

$$\max_{\{C_t,K_t,N_t\}_{t=0}^{\infty}} E[\sum_{t=1}^{\infty} \beta^t U(C_t, 1-N_t) + \lambda_t(Y_t + (1-\delta)K_{t-1} - C_t - K_t)]$$

$$\frac{\partial}{\partial C_t}: \quad \beta^t U'_C(C_t, 1-N_t) - \lambda_t = 0 \tag{1}$$

$$\frac{\partial}{\partial N_t}: \quad -\beta^t U'_{1-N}(C_t, 1-N_t) + \lambda_t \frac{\partial Y_t}{\partial N_t} = 0 \tag{2}$$

$$\frac{\partial}{\partial K_t}: \quad -\lambda_t + \lambda_{t+1}[\frac{\partial Y_{t+1}}{\partial K_t} + (1-\delta)] = 0 \tag{3}$$

- Combining equations (1) and (2)

$$U'_{1-N}(C_t, 1-N_t) = U'_C(C_t, 1-N_t)\frac{\partial Y_t}{\partial N_t}$$

this equation says that at the equilibrium the consumer is indifferent between having a leisure and get the utility of $U'_{1-N}(C_t, 1-N_t)$ and getting the utility of $U'_C(C_t, 1-N_t)$ from consuming each unit of good that s/he affords from producing $\partial Y_t/\partial N_t$, which, under competitive markets equal to the wage

- Combining equations (1) and (3)

$$U'_C(C_t, 1 - N_t) = \beta U'_C(C_{t+1}, 1 - N_{t+1}) R_{t+1}$$

where

$$R_t = \frac{\partial Y_t}{\partial K_{t-1}} + (1 - \delta)$$

the second equation is the gross return on capital. This is equal to the marginal product of capital, $\partial Y_t / \partial K_{t-1}$, plus the proportion of capital left at the end of capital $(1 - \delta)$. The left hand side of the first equation is marginal utility of consumption. This equation suggests that agents are indifferent between consuming today, or use it for saving and consume the return tomorrow

Example: Hansen's RBC Model

- The utility of a representative consumer

$$\max E[\sum_{t=1}^{\infty} \beta^t (\ln C_t + a(1 - N_t)]$$

subject to the resource constraint and dynamic equation for capital

$$C_t + I_t = Y_t \qquad K_t = I_t + (1 - \delta)K_{t-1}$$

production function

$$Y_t = Z_t K_{t-1}^{\rho} N_t^{1-\rho}$$

exogenous process for technology

$$\log Z_t = (1 - \psi) \log \bar{Z} + \psi \log Z_{t-1} + \epsilon_t$$

- In Lagrangian form

$$\max_{\{C_t,K_t,N_t\}_{t=0}^{\infty}} E[\sum_{t=1}^{\infty} \beta^t(\ln C_t + A(1-N_t)) + \lambda_t(Z_t K_{t-1}^{\rho} N_t^{1-\rho} + (1-\delta)K_{t-1} - C_t - K_t)]$$

$$\frac{\partial}{\partial C_t}: \quad \beta^t \frac{1}{C_t} - \lambda_t = 0 \tag{1}$$

$$\frac{\partial}{\partial N_t}: \quad -\beta^t A + \lambda_t(1-\rho)\frac{Y_t}{N_t} = 0 \tag{2}$$

$$\frac{\partial}{\partial K_t}: \quad -\lambda_t + \lambda_{t+1}[\rho \frac{Y_{t+1}}{K_t} + (1-\delta)] = 0 \tag{3}$$

  - Combining equations (1) & (2)

$$a = \frac{1}{C_t}(1-\rho)\frac{Y_t}{N_t} \tag{4}$$

– Combining equations (1) & (3)

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} R_{t+1} \tag{5}$$

where

$$R_t = \rho \frac{Y_t}{K_{t-1}} + (1 - \delta) \tag{6}$$

How can we use this model to analyze fluctuations in an economy?

1 First we need to find the steady state of the model that can explain the economy in its steady state. The economy is its steady state when there is no shock to economy. The model is its steady state when there is no shock to the model.

– As we eliminated growth from the model, the steady state of consumption satisfies that $C_t = C_{t+1} = ... = \bar{C}$. The case is similar for other variables.

* In this model there are many unknowns ($\bar{C}$, $\bar{Y}$, $\bar{K}$, $\bar{Z}$, $\bar{R}$, $\bar{I}$, $\bar{N}$, a, ρ, β, δ, σ). We can use the macro data to find some of these steady state values (the long-term average of variables) and use studies of micro data to calibrate some parameters. Then we can calculate the rest from the model

2 Then we should analyze the responses of the model to the shocks and see if the responses of the model match the fluctuations in the real economy. There are two common ways of doing that

2a Use growth accounting and find the technological progress in the data

$$\frac{\Delta Z}{\bar{Z}} = \frac{\Delta Y}{\bar{Y}} - \rho\frac{\Delta K}{\bar{K}} - (1-\rho)\frac{\Delta N}{N}$$

then supply economic model with these shocks (so called Solow Residuals) and see if it fits the data; i.e. if it matches the data and generates the same complex patterns (correlation among variables, autocorrelation and variance of each variable) with the actual data

2b Or (more common case) go directly to the economic model, and supply it with artificial shocks, ϵ_t. Then check if the generated data matches the second moments of the data (again correlation among variables, autocorrelation and variance of each variable)

3 Once you have a good model you have an idea of how the economy works. You can also use the model for policy analysis. For example you may change the parameters of the model and see how the economy (both its steady state and the way it responds to technological shocks) responds to that change

1- The Steady State

- Equation (4), (5) and (6) implies that

$$a = \frac{1}{\bar{C}}(1-\rho)\frac{\bar{Y}}{\bar{N}} \qquad 1 = \beta\bar{R} \qquad \bar{R} = \rho\frac{\bar{Y}}{\bar{K}}+(1-\delta) \quad \text{where} \quad \bar{Y} = \bar{Z}\bar{K}^{\rho}\bar{N}$$

  - To find some of A, $\bar{C}$, $\bar{Y}$, $\bar{K}$, $\bar{Z}$, $\bar{R}$, $\bar{I}$, $\bar{N}$, ρ, β, δ, σ, we can refer to the macro and micro data and use
  - $\bar{Z} = 1$ (Normalization)
  - $\rho = .36$ (Capital Share in the production)
  - $\bar{N} = 1/3$ (Steady state employment is a third of total time endowment)
  - $\delta = 0.025$ (Depreciation rate for capital)
  - $\bar{R} = 1.01$ (One percent real interest per quarter)

- $\psi = 0.95$ (Autocorrelation of technology shock)

- Then we can calculate the rest
  - $\beta = 1/\bar{R} = 0.99$
  - $\bar{Y}/\bar{K} = (\bar{R} + \delta - 1)/\rho$
  - $\bar{K} = (\bar{Y}/\bar{K} * 1/\bar{Z})\hat{}(1/(\rho - 1)) * \bar{N}$
  - $\bar{I} = \delta * \bar{K}$
  - $\bar{Y} = \bar{Y}/\bar{K} * \bar{K}$
  - $\bar{C} = \bar{Y} - \delta\bar{K}$
  - $a = 1/\bar{C} * (1 - \rho) * \bar{Y}/\bar{N}$ (Parameter in utility function)

- There is one final parameter left, but we are free to set it to any value
  - $\sigma = 0.712$; % Standard deviation of technology shock. Units: Percent

2 - Linearization around the steady state (I skip)

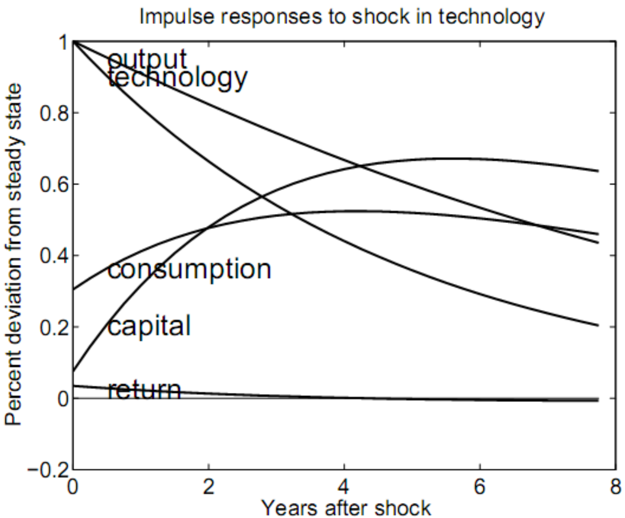
3 - Evaluating the Model

Standard deviations in percent (a) and correlations with output economies.

Series	Quarterly U.S. time series[a] (55,3–84,1)		Economy with divisible labor[b]	
	(a)	(b)	(a)	(b)
Output	1.76	1.00	1.35 (0.16)	1.00 (0.00)
Consumption	1.29	0.85	0.42 (0.06)	0.89 (0.03)
Investment	8.60	0.92	4.24 (0.51)	0.99 (0.00)
Capital stock	0.63	0.04	0.36 (0.07)	0.06 (0.07)
Hours	1.66	0.76	0.70 (0.08)	0.98 (0.01)
Productivity	1.18	0.42	0.68 (0.08)	0.98 (0.01)

- The first column shows the standard deviation of some variables in the US after they are detrended. The second column shows their correlation with output. The third and fourth columns show the same moments of the variables when they are generated from the model

4 - Impulse Responses from the Model and Policy Analysis



Impulse responses show us the effect of a temporary technology shock (ϵ_t) on the variables of the model, i.e. how they deviate from their steady state values. This figure is another way to see the information in the third and fourth column of the previous table

- Upon a temporary technology shock, output and marginal return to capital (and the interest rate) increase. This stimulates investment on capital. The increase in output induces people to consume more as well. After some periods, the economy goes back to its steady state

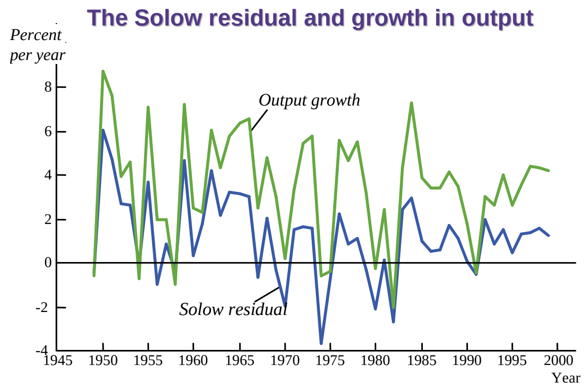
Why RBC?

- According to RBC theory
  - Economic fluctuations are caused by productivity shocks and they are optimal responses of variables (output, employment, and others) to these shocks. Shocks cause fluctuations not only in the output, but in the intertemporal wage as well. As a result workers respond them by adjusting labor supply. This causes employment, and through the employment, output to fluctuate as well
- The debate over RBC theory
  - Do changes in employment reflect voluntary changes in labor supply?

    - Critics argue that labor supply is not very sensitive to the intertemporal real wage, and high unemployment in recessions is mainly involuntary

- Does the economy experience large, exogenous productivity shocks in the short run?
- Are wages and prices flexible in the short run?
- Is money really neutral in the short run?
    - RBC critics note that reductions in money growth and inflation are almost always associated with periods of high unemployment and low output.
    - RBC proponents claims that the money supply is endogenous (if output is expected to fall, central bank reduces money supply for an expected fall in money demand)

- The Solow residual is a measure of productivity shocks: it shows the change in output that cannot be explained by changes in capital and labor (Y=F(K,AL)). RBC theory implies that the Solow residual should be highly correlated with output. Is it? Yes!



  – Critics note that since this residual is calculated from the output itself, one might expect it to be correlated with data