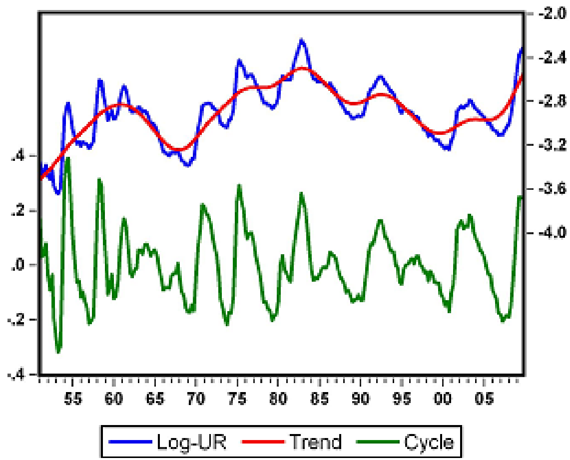
SEARCH AND MATCHING MODEL

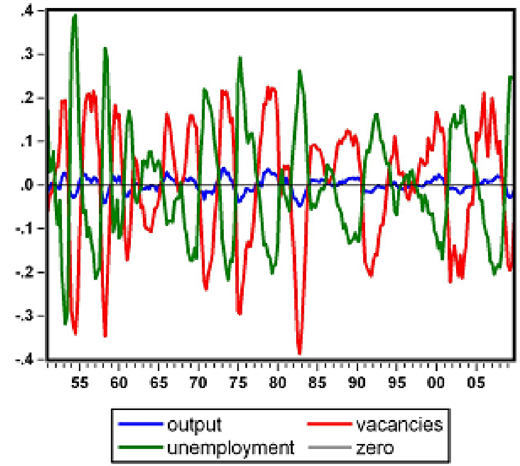
- The standard RBC model is not able to match the cyclical properties of every main variable in the economy
- Specifically, productivity shocks does not generate enough amplification and propagation of productivity shocks (that is, in the model the magnitude of the responds of variables to productivity shocks are weak, and these shocks do not diffuse enough throughout the model economy)
- In this chapter we will work over a model that is frequently integrated in RBC models to improve them
- This model is called Search and Matching Model. It analyses both the level and the cyclical properties of the unemployment rate in the economy. When it is integrated in RBC models, it is also able to improve the amplification and propagation of standard model in response to productivity shocks
- Here we do not use the model into RBC models, but only solve it

Main Questions: Take a look at Data

- What are the cyclical properties of the US labor market?

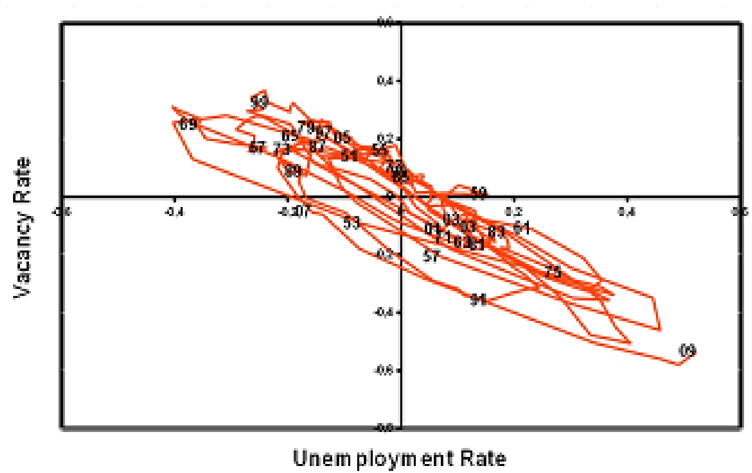
- The figure shows the level of the US unemployment rate. To obtain cyclical properties of the data, we need to decompose it to trend and cyclical components
- To do that the usual approach is to take log the data and apply the Hodrick-Prescott *filter*



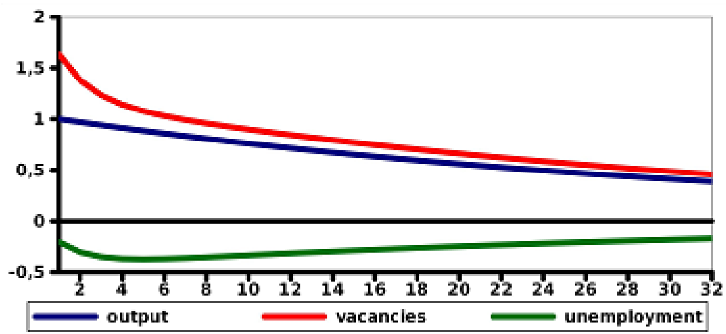


- The green line on the left shows the cyclicality of the US unemployment rate
- The figure on the right shows, inaddition, the cyclical part of number of vacancies (an empty job position), and the cyclical part of the output in the economy
- We see that vacancies are pro-cyclical, occuring probably because when output increases and economy expands firms have new job openings

- Unemployment is counter-cyclical (when there is more vacancies, people find jobs and the number of unemployed falls)
  - The correlation between vacancies and unemployment in the data is -0.99
  - A Beveridge curve is a graphical representation of this relationship



- There is another way of recognizing the same facts
  - The Model Impulse Response Functions are responses of model variables to the shocks in the model
  - One can also measure the effect of shocks on the real data (you need to take an *Applied Time Series* course!)



- Upon an increase in output, vacancies increase, unemployment falls
- Our question is: "Can the job matching model replicate this pattern?"

Model

- Unemployed workers search for a job
- Firms have vacancies (a position that is unfilled, unoccupied)
  - Firms and workers spend resources for job creation and production to happen
  - Then matching takes place between unemployed workers and jobs
  - There is no wage posting, instead wage determination occurs by bargaining
    * Baseline equilibrium model: Diamond-Mortensen-Pissarides (DMP) framework
- L : Labor force
- U : Unemployment
- u : Unemployment rate (U/L)
- V : Vacancies

- v : Vacancy Rate (per labor in the labor force: V/L)
  - uL number of unemployed workers and the vL number of job vacancies engage in matching

- The number of job matches taking place per unit time is given by

$$mL = m(uL, vL) \qquad \text{where} \;\; mL \leq \min(uL, vL)$$

this is the Matching Function, which increases in both its arguments, is concave, and homogeneous of degree 1. Matching function gives the number of jobs formed at any moment in time as a function of the number of workers looking for jobs and the number of firms looking for workers. In practice, not every matching is successful due to differences in the supply and demand for specific types of skills

  - The empirical literature finds that a log-linear (Cobb-Douglas) approximation to the matching function fits the data well ($mL = (uL)^{\alpha}(vL)^{1-\alpha}$)

– mL is the number of job matches taking place per unit time. The rate vacant is filled has the Poisson process

$$\frac{mL}{vL} = \frac{m(uL, vL)}{vL} = m(\frac{u}{v}, 1) = q(\theta) \qquad \text{where } \theta = \frac{v}{u}$$

θ is the indicator for tightness of the labor market. The higher the θ is (i.e. the higher the ratio of the vacancies (v) to unemployed workers (u)), the smaller $q(\theta)$ it results (the smaller the probability that a vacant job will be filled, which depends on u/v)

* During a small time interval δt, a vacant job is matched to an unemployed worker with probability $q(\theta)dt$, so the mean duration of a vacant job is

$$1/q(\theta)$$

– Similarly, unemployed workers move into employment according to the related Poisson process with rate

$$\frac{mL}{uL} = \frac{m(uL, vL)}{uL} = m(1, \frac{v}{u}), \quad \frac{mL}{uL} = \frac{m(uL, vL)}{uL} = \frac{m(uL, vL)}{vL}\frac{vL}{uL} = \theta q(\theta)$$

Since $m(1, v/u)$ is positively related with v/u, which equals θ, the higher the θ is (i.e. the higher the ratio of the vacancies (v) to unemployed workers (u)), the higher $\theta q(\theta)$ it results (the higher the probability that an unemployed worker will find a job)

* And the mean duration of unemployment is

$$1/\theta q(\theta)$$

  · Notice that there is a rationing in the model. $q(\theta)$ is decreasing in θ, but $\theta q(\theta)$ is increasing in that. So high θ is bad for firms, but good for workers

- How does the labor market work according to this model?
  - Job creation takes place when a firm and a unemployed worker meet and agree to form a match at a negotiated wage AEJ: Macroeconomics
  - Once the firm and worker meet and a job is created, production continues until a negative idiosyncratic shock arrives. These shocks results in job

destruction (employed workers become unemployed). Shocks occur at the Poisson rate λ

- The evolution of mean unemployment is, hence, given by

$$\dot{u} = \lambda(1-u) - \theta q(\theta)u$$

$\lambda(1-u)$ is the mean number of workers who enter unemployment during a small time interval, and $\theta q(\theta)u$ is the mean number of workers who leave unemployment

  – In the steady state the mean rate of unemployment is constant, so

$$\lambda(1-u) = \theta q(\theta)u$$

from this equation we can find that

$$u = \frac{\lambda}{\lambda + \theta q(\theta)} \tag{1}$$

which is to say that for given λ and θ, there is a unique equilibrium level of unemployment

Job Creation (Firm Problem)

- The number of jobs is endogenous and determined by profit maximization. Any firm is free to open a job vacancy and engage in hiring. Profit maximization requires that the profit from an additional vacancy should be zero

- Suppose a job's output is some constant $p > 0$. When the job is vacant, the firm is actively engaged in hiring at a fixed cost $pc > 0$ per unit time. During hiring, workers arrive to vacant jobs at the rate $q(\theta)$

- If J be the present-discounted value of expected profit from an occupied job, and V from an unoccupied job

  - In the infinite horizon, J and V satisfies the following Bellman equation

    $$rV = -pc + q(\theta)(J - V)$$

    taking $V = 0$ in the equilibrium condition for the supply of vacant jobs, this equation reduces to

    $$pc = q(\theta)J$$

- Interpretation of this equation is that the cost engaged in finding a suitable worker, $-pc$, is equal to the possibility that finding a suitable worker, $q(\theta)$, times the return from a return from a filled job, J. Hence,

$$J = \frac{pc}{q(\theta)} \tag{2a}$$

  Since $1/q(\theta)$ is the expected duration of a vacancy, another interpretation of this result is that in equilibrium the expected profit from a new job is equal to the expected cost of hiring a worker.

- J also satisfies

$$rJ = p - w - \lambda J \tag{2b}$$

  Yearly return from a filled job, rJ, for the firm is equal to the net return from a filled job (net return is $p - w$, where p is real output and w is the cost of labor, minus the possibility that an adverse shock, λ, times the loss it would cause, which is equal to a value of an existing match, J)

- Combining (2a) and (2b)

$$p - w = \frac{(r + \lambda)pc}{q(\theta)} \tag{2c}$$

Workers

- A typical worker earns w when employed, and searches for a job when unemployed. During search, s/he gets unemployment benefit z (where $w \geq z$)
- Let U and W denote the present-discounted value of the expected income stream of, respectively, an unemployed and an employed worker
  - For an unemployed labor:

$$rU = z + \theta q(\theta)(W - U) \tag{3a}$$

    The yearly value of being unemployed is equal to the unemployment benefit, z, plus the probability that s/he becomes employed, $\theta q(\theta)$, times the gain it would cause in worker's state

– For an employed labor:

$$rW = w + \lambda(U - W) \tag{3b}$$

The yearly value of being employed is equal to the wage, w, plus the probability s/he becomes unemployed, λ, times the loss it would cause in the worker's state

– Combining (3a) and (3b)

$$rU = \frac{[(r+\lambda)z + \theta q(\theta)w]}{r + \lambda + \theta q(\theta)} \tag{3c}$$

or

$$rW = \frac{\lambda z + [r + \theta q(\theta)w]}{r + \lambda + \theta q(\theta)} \tag{3c'}$$

which are the permanent incomes of unemployed and employed workers

Wage Determination (Bargain between Firms and Workers)

- Crucial condition is that matched agents (firm or worker) should be better of than searching agents. Thus, we need to look for reservation values of agents, the amount agents would accept to break up a match

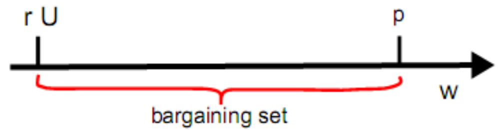
- Suppose a firm and a worker negotiates about the wage. For a successful match, both parties need to accept a common wage. This wage cannot be less than worker's reservation wage (the value of being jobless), but cannot be higher than worker's productivity, p, either. Otherwise firm would not find it optimal to hire worker. But what is worker's reservation wage?

  – Recall that for an unemployed worker

  $$rU = z + \theta q(\theta)(W - U) \tag{3a}$$

  this is also to say rU is the minimum compensation (wage) that an unemployed worker requires to give up search. This is the *reservation wage* of an unemployed, which s/he takes every period anyway

- Hence, a set for an acceptable wage is between is $[rU, p]$



  Parties negotiate for the wage that maximizes their well being. Since $p > rU$, a successful matching is possible

- The bargaining set contains infinitely many possible equilibrium wages. We need an additional assumptions to identify the wage that splits the mutual surplus:

$$W - U : \text{ the worker's surplus share} \quad J : \text{ the firm's surplus share} \tag{1}$$

  The standard assumption is *Nash Bargaining* over β

$$w = \arg\max_{w} (W(w) - U)^{\beta} J(w)^{1-\beta} \tag{4}$$

  - β is worker's bargaining power and $0 < \beta < 1$ (Symmetric Nash bargaining presumes $\beta = 0.5$)

- The first-order maximization condition derived from (4) satisfies that

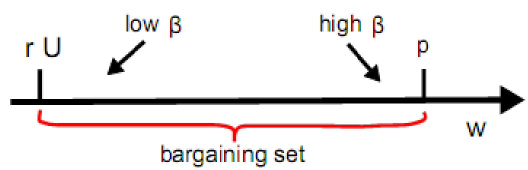
$$\frac{\beta}{(1-\beta)} = \frac{W-U}{J-V} \tag{5}$$

- Equations (5), (2b), (3b) and the condition V=0 find the flow version of equation (5)

$$w = rU + \beta(p - rU) \tag{6}$$

so workers get their reservation wage, rU, and a fraction β of the net surplus that they create by accepting the job. The above equation can be rearranged as

$$w = \beta p + (1-\beta)rU \tag{7}$$



- Equations (6), (2a), (3a) and (5) find the wage equation that holds in equilibrium

$$w = (1-\beta)z + \beta p(1 + c\theta)$$

Steady State Equilibrium

- Collecting 3 equations that determines the s.s. equilibrium for u, $\theta = v/u$, w

BC: The job flow condition (inflow equals outflow):

$$u = \frac{\lambda}{\lambda + \theta q(\theta)} \tag{1}$$

JC: The job creation condition of firms:

$$p - w = \frac{(r+\lambda)pc}{q(\theta)} \tag{2c}$$

(high w results in low v: higher wage reduces the incentives for job creation)

WC: The wage equation (Nash bargaining):

$$w = (1-\beta)z + \beta p(1 + c\theta) \tag{6}$$

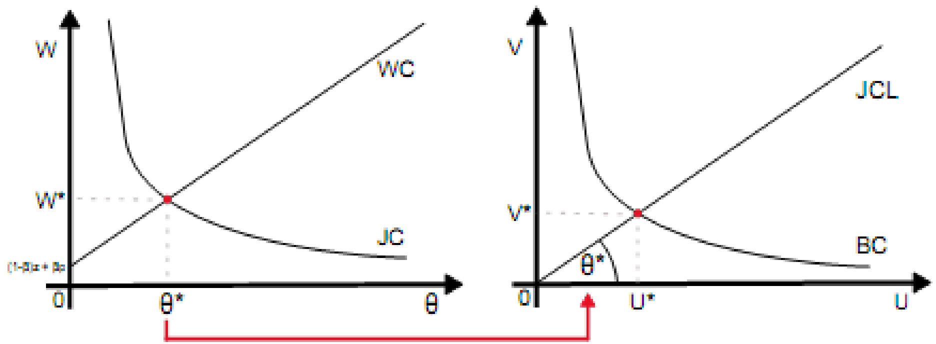
(tighter labor market (high θ), increases wage demand of workers)

- Equations (2c) and (7)determines wage rate (w) and (θ). Then we can use θ in equation (7) to determine u

- Rewriting the previous equations

$$BC : u = \frac{\lambda}{\lambda + \theta q(\theta)} \qquad JC : p - w = \frac{(r+\lambda)pc}{q(\theta)} \qquad WC : w = (1-\beta)z + \beta p(1 + c\theta)$$

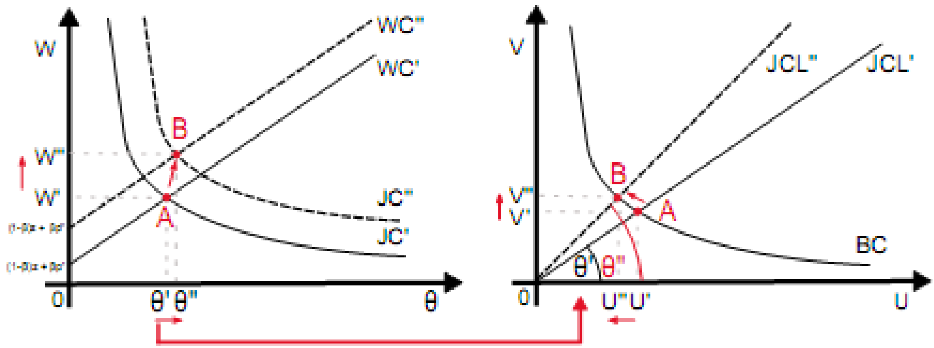
- Equilibrium can also be shown (to be unique) with the help of two diagrams



(the reason that Beveridge Curve is convex to the origin is the concavity of matching function ($mL = (uL)^{\alpha}(vL)^{\beta}$) in the input factors u and v)

An Example: The effect of an increase in productivity (from p' to p'') on w, u, v

$$BC : u = \frac{\lambda}{\lambda + \theta q(\theta)} \qquad JC : p - w = \frac{(r+\lambda)pc}{q(\theta)} \qquad WC : w = (1-\beta)z + \beta p(1 + c\theta)$$



The rise in productivity shifts WC and JC upwards
Wage rises (equation WC)
Labor market tightness (θ) may rise or not depending on the value of β and c
If θ rises: JCL rotates anti clockwise: Vacancies rise, unemployment falls
If θ declines: JCL rotates clockwise: Vacancies falls, unemployment rises