TOBB-ETU, Economics Department

Applied Time Series (IKT 553)

2020/21 - HW 1 (Ozan Eksi)

Each question is 20 points.

1-) For each of the following, determine whether $\{y_t\}$ is a stable process by using characteristic roots of the homogenous part of equaion. Next, write each of the above equations using lag operators. Use the characteristic roots of the lag polynomial (which is called the inverse characteristic equation) to determine whether $\{y_t\}$ is a stable process or not.

a-)

$$y_t + 0.7y_{t-1} - 0.3y_{t-2}$$

b-)

$$y_t - 1.2y_{t-1} - 1.2y_{t-2}$$

c-)

$$y_t + 0.8y_{t-1}$$

2-) Suppose that the money supply process has the form $m_t = c + \rho m_{t-1} + \varepsilon_t$ where c is a constant and $0 < \rho < 1$.

a-) Show that it is possible to express m_{t+n} in terms of a constant term, the known value m_t and the sequence $\{\varepsilon_{t+1}, \varepsilon_{t+2}, \ldots, \varepsilon_{t+n}\}$.

b-) Suppose that all values of ε_{t+i} for $i > 0$ have a mean value of zero. Use your result in part a-) to forecast the money supply n-periods into the future and as time goes to infinity.

c-) Suppose $m_1 = 10$, $c = 2$ and $\rho = 0.5$. Use e-views to plot the next 150 realizations of the series above. Next, do the same thing by assuming $c = 0$? Interpret your findings.

3-) Use the following MA process for the following questions:

$$y_t = 1/2\varepsilon_t + 1/2\varepsilon_{t-1} + 1/2\varepsilon_{t-2} + 1/2\varepsilon_{t-3}$$

where ε_t's are white noise processes with variance σ^2

a-) Find the expected value of y_t.

b-) Find the expected value of y_t given that $\varepsilon_{t-3} = \varepsilon_{t-2} = 1$.

c-) Find $var(y_t)$

d-) Find $cov(y_t, y_{t-2})$

4-) Consider the second-order autoregressive process

$$y_t = a_0 + a_2 y_{t-2} + \varepsilon_t,$$

where $\mid a_2 \mid < 1$. Find $E_{t-2}(y_t)$, $E_{t-1}(y_t)$, $var(y_t)$, $cov(y_t, y_{t-1})$ and $cov(y_t, y_{t-2})$. (Hint: You may use Yule-Walker conditions to calculate the last three measures.)

5-) The file HW2_data.xlsx contains the U.S. data. Form the spread, s_t, by subtracting the t-bill rate from the 5-year rate.

a-) One difficulty in performing a unit root test is to select the proper lag length. Estimate model of the form

$$\Delta s_t = a_0 + \gamma s_{t-1} + \sum \beta_i \Delta s_{t-i}$$

Use the AIC and SIC methods to select the appropriate lag length. You should find that the AIC and SIC methods select lag lengths of 9 and l, respectively. In this case, does the lag length matter for the Dickey–Fuller test?

b-) Use a lag length of 8 and perform an augmented Dickey-Fuller test of the spread. You should find

$$\Delta s_t = 0.255 - 0.211 s_{t-1} + \sum \beta_i \Delta s_{t-i}$$

Is the spread stationary?

c-) Perform an augmented Dickey-Fuller test of the 5-year rate using seven lags. Is the 5-year rate stationary?

d-) Perform an augmented Dickey-Fuller test of the t-bill rate using 11 lags. Is the t-bill rate stationary?

e-) How is it possible that the individual rates (t-bill and 5-year rates) act as I(1) processes whereas the spread acts as a stationary process?