

Appendix 1

Formulas for Calculating Chain-Type Quantity and Price Indexes

This appendix shows the basic calculations used to prepare annual and quarterly chain-type quantity and price indexes.

Annual indexes

The formula used to calculate the annual change in real GDP and other components of output and expenditures is a Fisher index (Q_t^F) that uses weights for 2 adjacent years (years $t-1$ and t).

The formula for real GDP in year t relative to its value in year $t-1$ is

$$Q_t^F = \sqrt{\frac{\sum p_{t-1} q_t}{\sum p_{t-1} q_{t-1}} \times \frac{\sum p_t q_t}{\sum p_t q_{t-1}}}$$

where the p 's and q 's represent prices and quantities of detailed components in the 2 years.

Because the first term in the Fisher formula is a Laspeyres quantity index (Q_t^L), or

$$Q_t^L = \frac{\sum p_{t-1} q_t}{\sum p_{t-1} q_{t-1}}$$

and the second term is a Paasche quantity index (Q_t^P), or

$$Q_t^P = \frac{\sum p_t q_t}{\sum p_t q_{t-1}}$$

the Fisher formula can also be expressed for year t as the geometric mean of these indexes as follows:

$$Q_t^F = \sqrt{Q_t^L \times Q_t^P}$$

The percent change in real GDP (or in a GDP component) from year $t-1$ to year t is calculated as

$$100 (Q_t^F - 1.0).$$

Similarly, price indexes are calculated using the Fisher formula

$$P_t^F = \sqrt{\frac{\sum p_t q_{t-1}}{\sum p_{t-1} q_{t-1}} \times \frac{\sum p_t q_t}{\sum p_{t-1} q_t}}$$

which is the geometric mean of a Laspeyres price index (P_t^L) and a Paasche price index (P_t^P), or

$$P_t^F = \sqrt{P_t^L \times P_t^P}$$

The chain-type quantity index value for period t is $I_t^F = I_{t-1}^F \times Q_t^F$, and the chain-type price index is calcu-

lated analogously. Chain-type real output and price indexes are presented with the reference year (b) equal to 100; that is, $I_b = 100$.

The current-dollar change from year $t-1$ to year t expressed as a ratio is equal to the product of the Fisher price and quantity indexes:

$$\frac{\sum p_t q_t}{\sum p_{t-1} q_{t-1}} = \sqrt{\frac{\sum p_t q_{t-1}}{\sum p_{t-1} q_{t-1}} \times \frac{\sum p_t q_t}{\sum p_{t-1} q_t}} \times \sqrt{\frac{\sum p_{t-1} q_t}{\sum p_{t-1} q_{t-1}} \times \frac{\sum p_t q_t}{\sum p_t q_{t-1}}} = P_t^F \times Q_t^F$$

Quarterly indexes

The same formulas are used to calculate the quarterly indexes except that quarterly data are substituted for annual data.

All quarterly chain-type indexes for completed years that have been included in an annual or comprehensive revision are adjusted so that the quarterly indexes average to the corresponding annual index. When an additional year is completed between annual revisions, the annual index is computed as the average of the quarterly indexes, so no adjustment is required to make the quarterly and annual indexes consistent. For example, until the 2006 annual revision was released, the chain-type indexes for the year 2005 were computed as the average of the four quarterly indexes for 2005.

Chained-dollar estimates

The chained-dollar value CD_t^F is calculated by multiplying the index value by the reference year current-dollar value ($\sum p_b q_b$) and dividing by 100.¹ For period t ,

$$CD_t^F = \sum p_b q_b \times I_t^F / 100.$$

Implicit price deflators

The implicit price deflator IPD_t^F for period t is calculated as the ratio of the current-dollar value to the corresponding chained-dollar value, multiplied by 100, as follows:

$$IPD_t^F = \frac{\sum p_t q_t}{CD_t^F} \times 100.$$

1. For exceptions to this procedure, see appendix 2.