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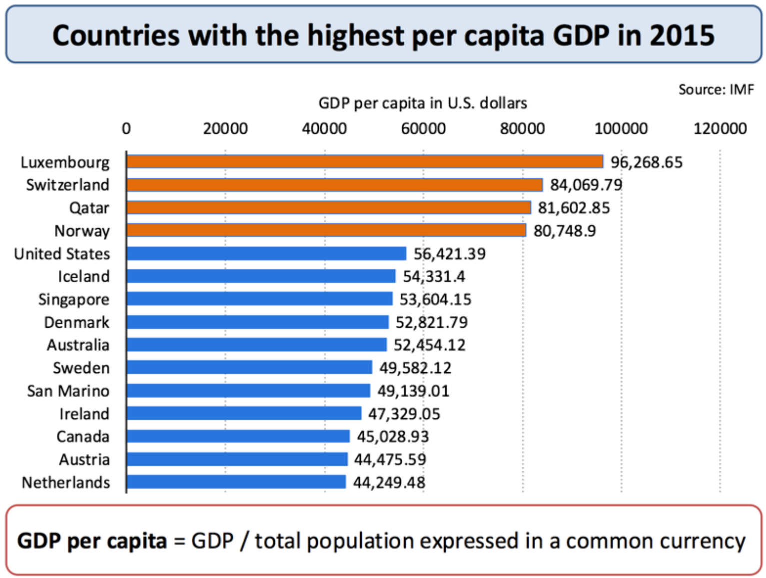
## *Part 3 - GROWTH THEORY: THE ECONOMY IN THE VERY LONG RUN*

Two Main Assumptions:

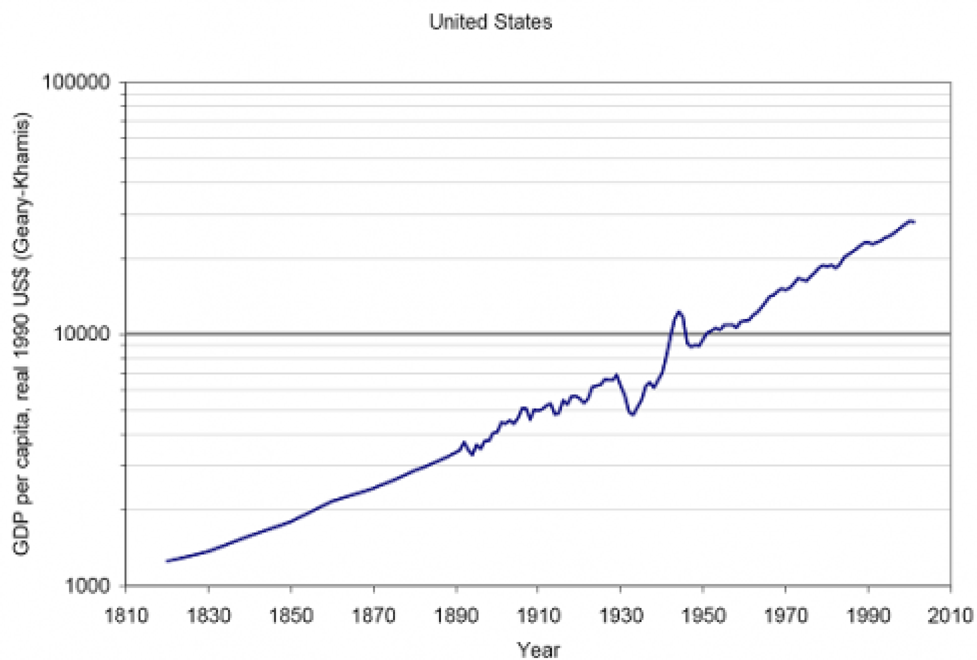
- Prices are flexible
- Economic growth comes from labor, technology and capital

## Ch7 - Economic Growth I

- Income differences across countries



- Income growth over time



- *Economic Growth* is the branch of economics attempts to explain increases in national income
- Capital accumulation story is appropriate for underdeveloped nations (that is called *Economic Development*)
- Developed countries are assumed to have reached their steady state of (per capita) capital, as a result technological developments are more appropriate to explain growth across these countries
- Nearly all growth models assumes that labor is fully employed and it grows at a constant rate; hence, they abstract from labor market dynamics

## SOLOW-SWAN MODEL

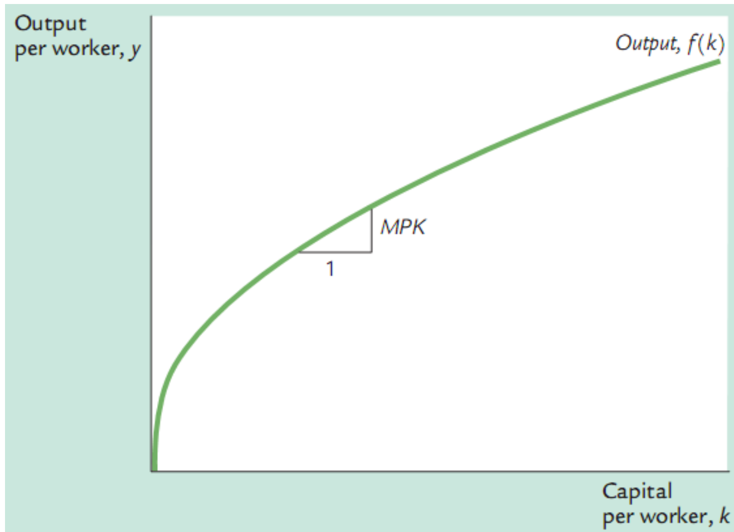
- The Solow model assumes exogenous technological progress and saving rate
- The model shows how an economy converges to its steady state
  - Steady state condition defines the situation where macroeconomic variables grow at constant rate

**Production Function**

- The supply of goods is found from production function: $Y = F(K, L)$
- This also can be written as (due to the CRTS property): $Y/L = F(K/L, 1)$
- It states that output per worker is a function of capital per worker
- If y=Y/L and k=K/L, then: $y = f(k)$
- Notice that f shows DRTS property

- Ex:

$$Y = F(K, L) = K^{1/2}L^{1/2} \quad \Rightarrow \quad y = k^{1/2}$$



**The Demand for Goods**

- The demand for goods comes from consumption and investment: $y = c + i$

**The Change in the Capital Stock**

- Each year people save a fraction s of their income and consume a fraction (1-s)

$$y = (1 - s)y + i \quad \Rightarrow \quad i = sy = sf(k)$$

- Hence, per capira consumption is given by

$$y = c + sy \quad \Rightarrow \quad c = (1 - s)y$$

- Change in Capital Stock = Investment - Depreciation

$$\dot{K} = sF(K, L) - \delta K$$

can be written in terms of $k$ and $\dot{k}$, where $\dot{k}$ can be found by

$$\dot{k} = (\frac{\dot{K}}{L}) = \frac{\dot{K}L - K\dot{L}}{L^2} = \frac{\dot{K}}{L} - \frac{K}{L}\frac{\dot{L}}{L}$$

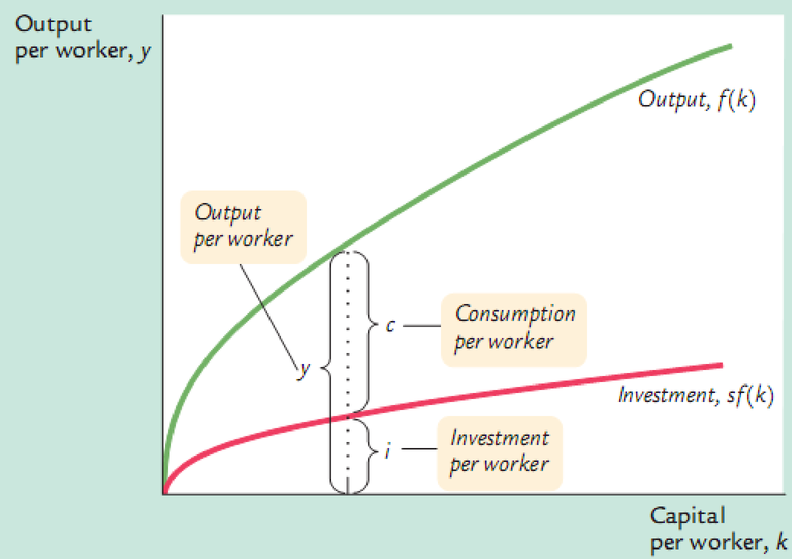
if $\dot{L}/L = 0$

$$\dot{k} = \frac{sF(K, L)}{L} - \delta k = sf(k) - \delta k$$

that is, in discrete time

$$\Delta k = sf(k) - \delta k$$

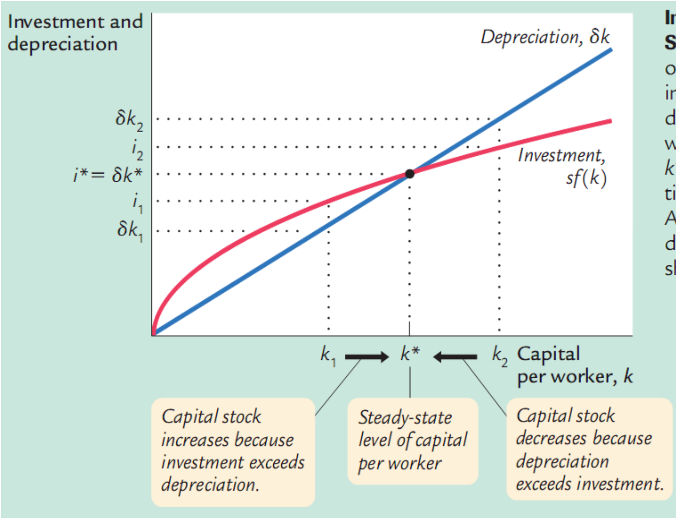
$$\Delta k = sf(k) - \delta k$$




**Output, Consumption, and Investment** The saving rate $s$ determines the allocation of output between consumption and investment. For any level of capital $k$, output is $f(k)$, investment is $sf(k)$, and consumption is $f(k) - sf(k)$.

### The Steady State

At the steady state, investment equals depreciation. Hence, $\Delta k^* = 0$ & $sf(k^*) = \delta k^*$




**Investment, Depreciation, and the Steady State** The steady-state lev of capital $k^*$ is the level at which investment equals depreciation, in dicating that the amount of capit will not change over time. Below $k^*$, investment exceeds depreciation, so the capital stock grows. Above $k^*$, investment is less than depreciation, so the capital stock shrinks.

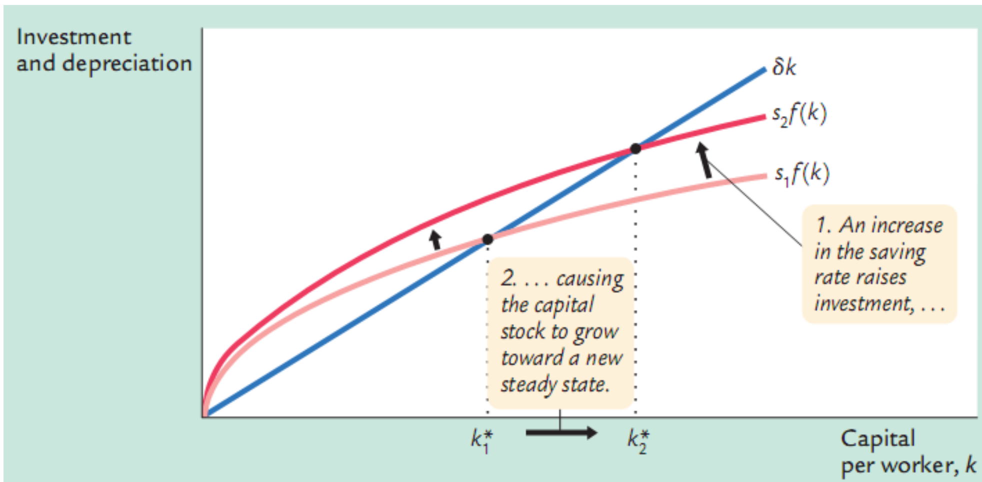
- Regardless of the level of capital with which the economy begins, it ends up with the steady-state level of per capita capital, $k^*$

| Variable | Symbol | S. S. Growth Rate |
|---|---|---|
| Output per Worker | $y = f(k)$ | 0 |
| Output | $Y = y * L$ | $n$ |

where $n$ can be 0 if the population growth rate is 0

**How Saving Affects Growth**

- If the saving rate is high, the economy will have a large capital stock and a high level of output. If the saving rate is low, on the contrary



- The rapid growth in Japan and Germany is what the Solow model predicts for countries in which war has greatly reduced the capital stock, which was followed by periods of high saving rates

**Dynamic Equation for Capital Stock**

The following dynamic equation

$$\dot{K} = sF(K, L) - \delta K$$

can be written in terms of $k$ and $\dot{k}$, where $\dot{k}$ can be found by

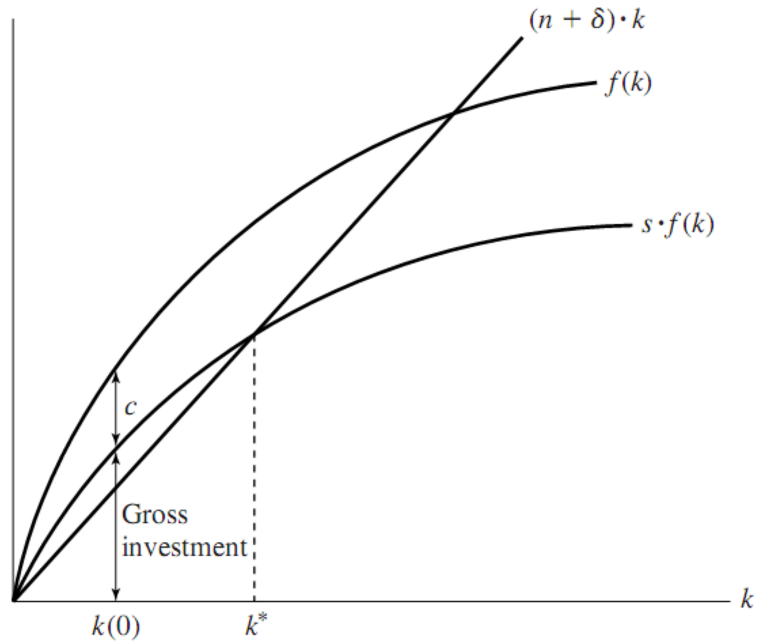
$$\dot{k} = (\frac{\dot{K}}{L}) = \frac{\dot{K}L - K\dot{L}}{L^2} = \frac{\dot{K}}{L} - \frac{K}{L}\frac{\dot{L}}{L}$$

if $\dot{L}/L = 0$

$$\dot{k} = \frac{sF(K, L)}{L} - \delta k - nk$$

$$\Rightarrow \dot{k} = sf(k) - (\delta + n)k$$

where $(\delta + n)$ is the effective depreciation rate



$(n + \delta) \cdot k$
$f(k)$
$s \cdot f(k)$
$c$
Gross investment
$k$
$k(0)$
$k^*$

**The Golden Rule Level of Capital**

- The steady state level of consumption $c^* = (1-s)f(k^*)$
- The saving rate that maximizes the steady state consumption per person is called the *Golden Rule Level of Saving Rate*, and denoted by

$$s_{gold} : \max_s c^* = \max_s (1-s)f(k^*(s))$$

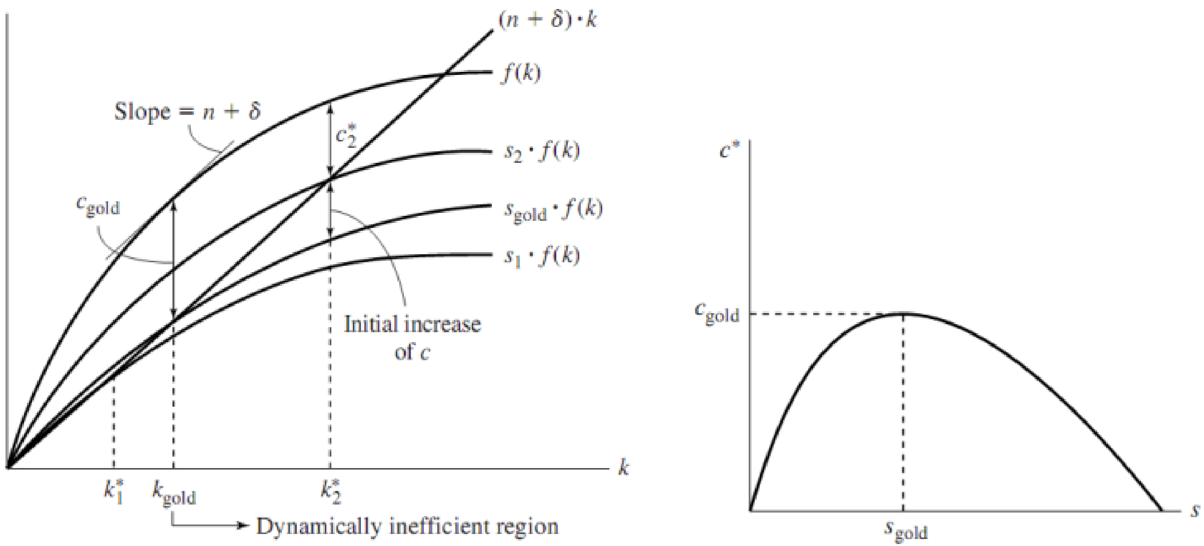
  – We have shown that at the steady state $sf(k^*) = (\delta + n)k^*$.

– This implies that

$$\max_{s} c^* = \max_{k^*}[f(k^*) - (\delta + n)k^*]$$

FOC requires that

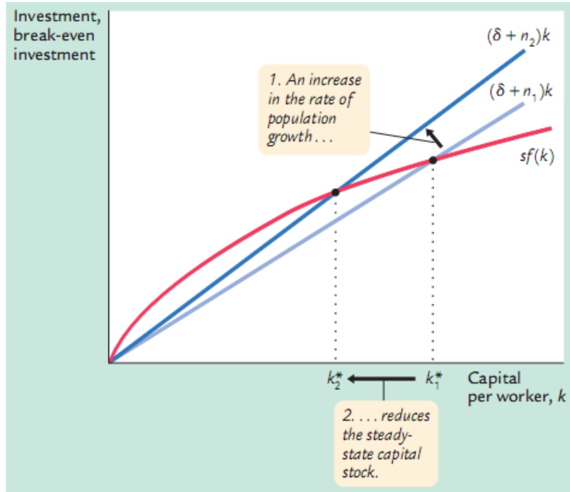
$$f'(k_{gold}) = (\delta + n)$$




- $f'(k_{gold}) = (\delta + n)$ implies that it is optimal to accumulate capital till its marginal product equals to the effective depreciation rate. Before (after) this point the marginal product of capital is higher (lower) than the effective depreciation rate.

- If $s > s_{gold}$, it is dynamically inefficient region ($k^* > k_{gold}$ but $c^* < c_{gold}$!)

### The Effect of Population Growth




- As $k^*$ is lower, output per worker $y^* = f(k^*)$ is also lower
- Thus, the Solow model predicts that countries with higher population growth will have lower levels of GDP per person

**Ch8 - Economic Growth II-Technological Progress in the Solow Model**

- The new production function

$$Y = F(K, L * E)$$

  where E is called the efficiency of labor that increases with technology

  - The term L*E measures the number of effective workers
  - E grows at some constant rate g
  - As the labor force L is growing at rate n, the number of effective workers L * E is growing at rate n + g

| Variable | Symbol | S. S. Growth Rate |
| --- | --- | --- |
| Output per Worker | $y = f(k)$ | $g$ |
| Output | $Y = y * L$ | $n + g$ |

**The Steady State With Technological Progress**

- Let $\hat{k} = K/(L * E)$ stand for capital per effective worker and $\hat{y} = Y/(L * E)$ stand for output per effective worker. With these definitions,we can write

$$\Delta\hat{k} = sf(\hat{k}) - (\delta + n + g)\hat{k}$$

  - where $gk$ is needed to provide capital for the new "effective workers"

- Proof:

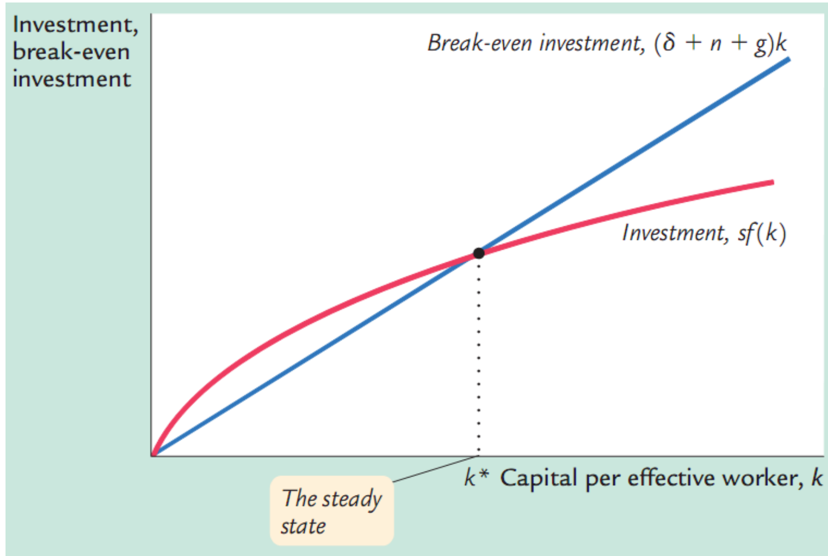
$$\dot{K} = sF(K, L) - \delta K$$

$$\hat{k}^{\cdot} = (\frac{\dot{K}}{LE}) = \frac{\dot{K}EL - KE\dot{L} - K\dot{E}L}{E^2L^2} = \frac{\dot{K}}{EL} - \frac{K}{EL}\frac{\dot{L}}{L} - \frac{K}{EL}\frac{\dot{E}}{E}$$

$$\hat{k}^{\cdot} = \frac{sF(K, L) - \delta K}{EL} - n\hat{k} - g\hat{k}$$

$$\Rightarrow \hat{k}^{\cdot} = sf(\hat{k}) - (\delta + n + g)\hat{k}$$

- The analysis is very similar to the previous one



| Variable | Symbol | S. S. Growth Rate |
|---|---|---|
| Output per Eff. Worker | $\hat{y} = f(\hat{k})$ | $0$ |
| Output per Worker | $y = f(k) = \hat{y} * E$ | $g$ |
| Output | $Y = y * L$ | $n + g$ |

**Types of Capital**

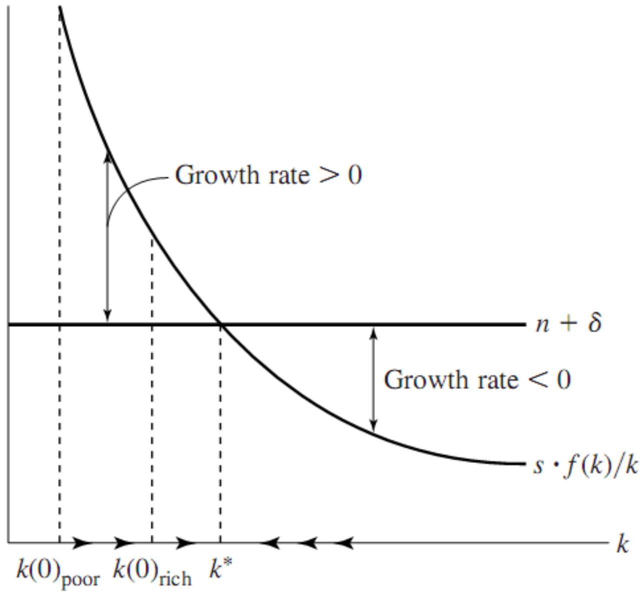
- The Solow model makes the simplifying assumption that there is only one type of capital. In the world, there are many types
  - The government invests in various forms of public capital, called infrastructure, such as roads, bridges, and sewer systems
  - In addition, there is human capital—the knowledge and skills that workers acquire through education

**Encouraging Technological Progress**

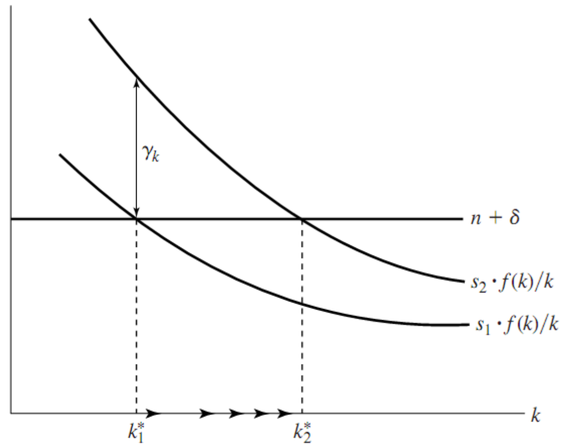
- The Solow model takes technological progress as exogenous
- Yet, policies encourage the private sector to devote resources to technological innovation. For example
  - the patent system; subsidizing basic research in universities
  - promoting specific industries that are key for rapid technological progress (Industrial Policy)
- Sometimes new ideas become part of society' s pool of knowledge (knowledge spillover). Such a by-product is called a technological externality. In this case, the social returns to capital exceed the private returns

**Transitional Dynamics**

- $\gamma_k = \frac{\dot{k}}{k} = s\frac{f(k)}{k} - (\delta + n) \quad \gamma_k$ : the capital growth rate


Growth rate > 0
$n + \delta$
Growth rate < 0
$s \cdot f(k)/k$
$k$
$k(0)_{poor}$
$k(0)_{rich}$
$k^*$

**Policy Experiment: Effects of an increase in the saving rate**




- A permanent increase in saving rate generates a temporarily positive per capita growth rates

**Absolute and Conditional Convergence**

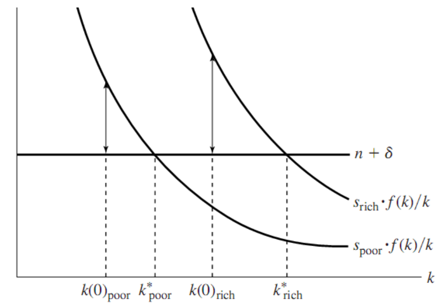
Smaller values of $k$ are associated with larger values of $\gamma_k$

Question: Does this mean that economies of lower capital per person tend to grow faster in per capita terms? Does there tend to be convergence across economies?

- *Absolute Convergence:* The hypothesis that poor countries grows faster per capita than rich ones without conditioning on any other characteristics of the economy
- *Conditional Convergence:* If one drops the assumption that all economies have same parameters, and thus same steady state values, then an economy grows faster the fur-

ther it is own steady state value

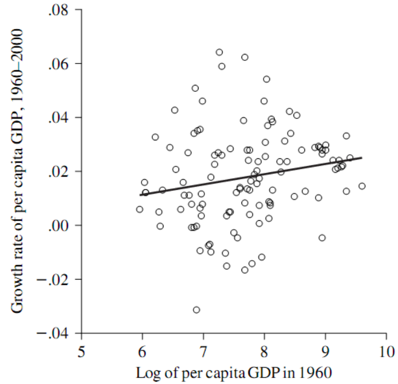
- Ex: consider two economies that differ only in two respects: $s_{rich} > s_{poor}$ & $k(0)_{rich} > k(0)_{poor}$
  - Does the model predict poor economy will grow faster than the rich one?
    - * Not necessarily. See the figure:



∗ Data:

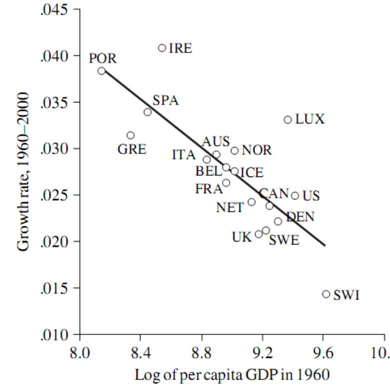
Absolute Convergence

Sample of 114 countries




Conditional Convergence

Sample of OECD countries




- The economies of the world exhibit conditional convergence: they appear to be converging to their own steady states, which in turn are determined by saving, population growth, and education

**A Note on World Income Distribution**

- Two seemingly puzzling facts
  - World Income Inequality that is found by integrating individuals' income distribution over large sample of countries has declined between 1970s and 2000s
  - Within country income inequalities has increases over the same period
    - Explanation: Some of the poorest and most populated countries in the World (most notably China and India, but also many other countries in Asia) rapidly *converged* to the incomes of OECD citizens

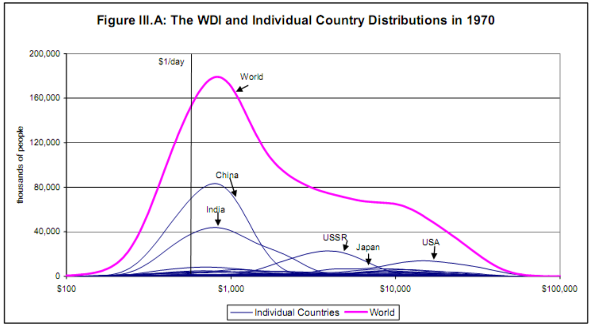


Figure III.A: The WDI and Individual Country Distributions in 1970
200,000
160,000
120,000
80,000
40,000
0
thousands of people
$1/day
World
China
India
USSR
Japan
USA
$100
$1,000
$10,000
$100,000
Individual Countries
World

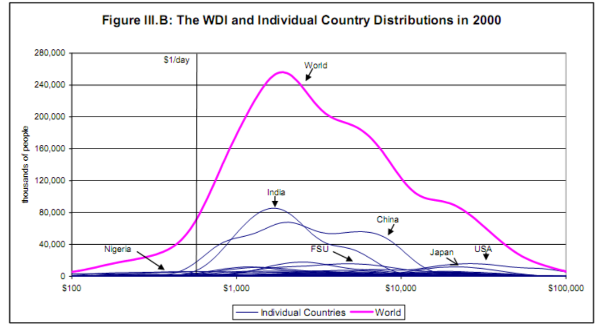


Figure III.B: The WDI and Individual Country Distributions in 2000
280,000
240,000
200,000
160,000
120,000
80,000
40,000
0
thousands of people
$1/day
World
India
China
Nigeria
FSU
Japan
USA
$100
$1,000
$10,000
$100,000
Individual Countries
World

**APPENDIX: Growth Accounting (that you are not responsible from)**

- Growth accounting divides the growth in output into three different sources: increases in capital, increases in labor, and advances in technology

- The production function with technological process can be written as

$$Y = AF(K, L)$$

where A is a measure of the current level of technology called total factor productivity

- The growth accounting equation can be written as

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

- or

$$\frac{\Delta A}{A} = \frac{\Delta Y}{Y} - \alpha \frac{\Delta K}{K} - (1-\alpha)\frac{\Delta L}{L}$$

  $\Delta A/A$ is sometimes called the Solow residual, after Robert Solow, who first showed how to compute it

- One can show that $\Delta A/A = (1-\alpha)\Delta E/E$, where a is capital' s share. Thus, technological change as measured by growth in the efficiency of labor is proportional to technological change as measured by the Solow residual

- Total factor productivity captures anything that changes the relation between measured inputs and measured output
- It is used by Classical Economists to explain the changes in Real GDP. This view implies that changes in supply cause economic fluctuations

**table 8-3**

**Accounting for Economic Growth in the United States**

| Years | Output Growth $\Delta Y/Y$ | = | SOURCE OF GROWTH<br>Capital $\alpha\Delta K/K$ | + | Labor $(1-\alpha)\Delta K/K$ | + | Total Factor Productivity $\Delta A/A$ |
|---|---|---|---|---|---|---|---|
| | | | (average percentage increase per year) | | | | |
| 1950–1999 | 3.6 | | 1.2 | | 1.3 | | 1.1 |
| 1950–1960 | 3.3 | | 1.0 | | 1.0 | | 1.3 |
| 1960–1970 | 4.4 | | 1.4 | | 1.2 | | 1.8 |
| 1970–1980 | 3.6 | | 1.4 | | 1.2 | | 1.0 |
| 1980–1990 | 3.4 | | 1.2 | | 1.6 | | 0.6 |
| 1990–1999 | 3.7 | | 1.2 | | 1.6 | | 0.9 |

*Source:* U.S. Department of Commerce, U.S. Department of Labor, and the author's calculations. The parameter $\alpha$ is set to equal 0.3.