Practice Problems 3

Problem 1: Hansen's RBC Model (from lecture notes)

We define an economy where the utility of a representative consumer

$$\max E[\sum_{t=1}^{\infty} \beta^t (\ln C_t + a(1 - N_t)]$$

is subject to the resource constraint

$$C_t + K_t = Y_t + (1 - \delta) K_{t-1}$$

The production function is given as

$$Y_t = Z_t K_{t-1}^{\rho} N_t^{1-\rho}$$

and exogenous process for technology:

$$\log Z_t = (1 - \psi) \log \bar{Z} + \psi \log Z_{t-1} + \epsilon_t$$

a-) Solve the social planner's problem and find the equilibrium conditions of the economy

Solution:

The problem in Lagrangian form

$$\max_{\{C_t,K_t,N_t\}_{t=0}^{\infty}} E[\sum_{t=1}^{\infty} \beta^t(\ln C_t + a(1-N_t)) + \lambda_t(Z_t K_{t-1}^{\rho} N_t^{1-\rho} + (1-\delta)K_{t-1} - C_t - K_t)]$$

$$\frac{\partial}{\partial C_t} : \quad \beta^t \frac{1}{C_t} - \lambda_t = 0 \tag{1}$$

$$\frac{\partial}{\partial N_t} : \quad -\beta^t a + \lambda_t (1-\rho)\frac{Y_t}{N_t} = 0 \tag{2}$$

$$\frac{\partial}{\partial K_t} : \quad -\lambda_t + \lambda_{t+1}[\rho \frac{Y_{t+1}}{K_t} + (1-\delta)] = 0 \tag{3}$$

Combining equations (1) and (2)

$$a = \frac{1}{C_t}(1-\rho)\frac{Y_t}{N_t} \tag{4}$$

Combining equations (1) and (3)

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} R_{t+1} \tag{5}$$

where

$$R_t = \rho \frac{Y_t}{K_{t-1}} + (1-\delta) \tag{6}$$

b-) Find the steady state values of consumption, capital, investment, output, consumption ($\bar{K}$, $\bar{I}$, $\bar{Y}$, $\bar{C}$), and also the parameter in utility function for leisure (a) given that the steady state value of technological process is 1 ($\bar{Z} = 1$), the capital share in the production is 0.36, steady state employment is a third of total time endowment ($\bar{N} = 1/3$), depreciation rate for capital is 0.025, the long term average of the real interest per quarter (gross) is 1.01

Solution: Equations (4), (5) and (6) implies that

$$a = \frac{1}{\bar{C}}(1-\rho)\frac{\bar{Y}}{\bar{N}} \qquad 1 = \beta \bar{R} \qquad \bar{R} = \rho \frac{\bar{Y}}{\bar{K}} + (1-\delta) \;\; \text{where} \quad \bar{Y} = \bar{Z}\bar{K}^{\rho}\bar{N}^{1-\rho}$$

We have given that $\bar{Z} = 1$, $\rho = .36$, $\bar{N} = 1/3$, $\delta = 0.025$, $\bar{R} = 1.01$

Then we can calculate the rest of the parameters and steady state values of variables

$\beta = 1/\bar{R} = 0.99$

$\bar{Y}/\bar{K} = (\bar{R} + \delta - 1)/\rho = 0.097$

$\bar{K} = (\bar{Y}/\bar{K} * 1/\bar{Z})\hat{}(1/(\rho - 1)) * \bar{N} = 12.72$

$\bar{I} = \delta * \bar{K} = 0.318$

$\bar{Y} = \bar{Y}/\bar{K} * \bar{K} = 1.237$

$\bar{C} = \bar{Y} - \delta\bar{K} = 0.919$

$a = 1/\bar{C} * (1 - \rho) * \bar{Y}/\bar{N} = 2.58$

c-) Increase $\bar{Z}$ to 1.1 and find the new steady states of $(\bar{K}, \bar{I}, \bar{Y}, \bar{C})$. Then give an economic interpretation to changes in the variables from their initial values

Solution: $a = \frac{1}{\bar{C}}(1-\rho)\frac{\bar{Y}}{\bar{N}} \quad 1 = \beta\bar{R} \quad \bar{R} = \rho\frac{\bar{Y}}{\bar{K}} + (1-\delta)$ where $\bar{Y} = \bar{Z}\bar{K}^{\rho}\bar{N}^{1-\rho}$

We have given that $\bar{Z} = 1$, $\rho = .36$, $\bar{N} = 1/3$, $\delta = 0.025$, $\bar{R} = 1.01$, then

$\beta = 1/\bar{R} = 0.99$

$\bar{Y}/\bar{K} = (\bar{R} + \delta - 1)/\rho = 0.097$ (previously it was 0.097)

$\bar{K} = (\bar{Y}/\bar{K} * 1/\bar{Z})\hat{}(1/(\rho - 1)) * \bar{N} = 14.76$ (previously it was 12.72)

$\bar{I} = \delta * \bar{K} = 0.369$ (previously it was 0.318)

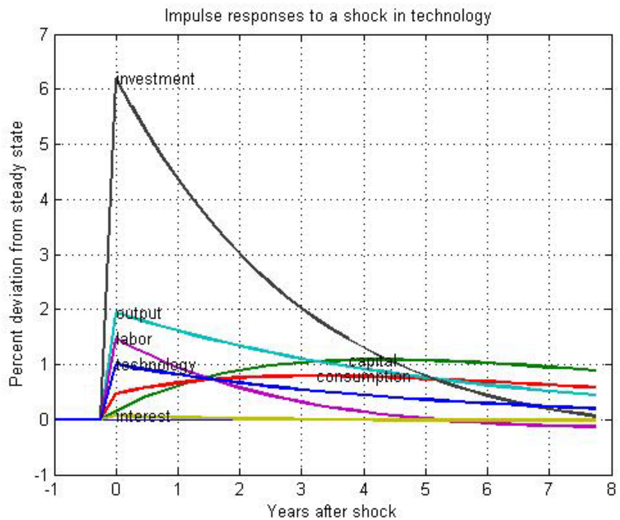
$\bar{Y} = \bar{Y}/\bar{K} * \bar{K} = 1.435$ (previously it was 1.237)

$\bar{C} = \bar{Y} - \delta\bar{K} = 1.066$ (previously it was 0.91)

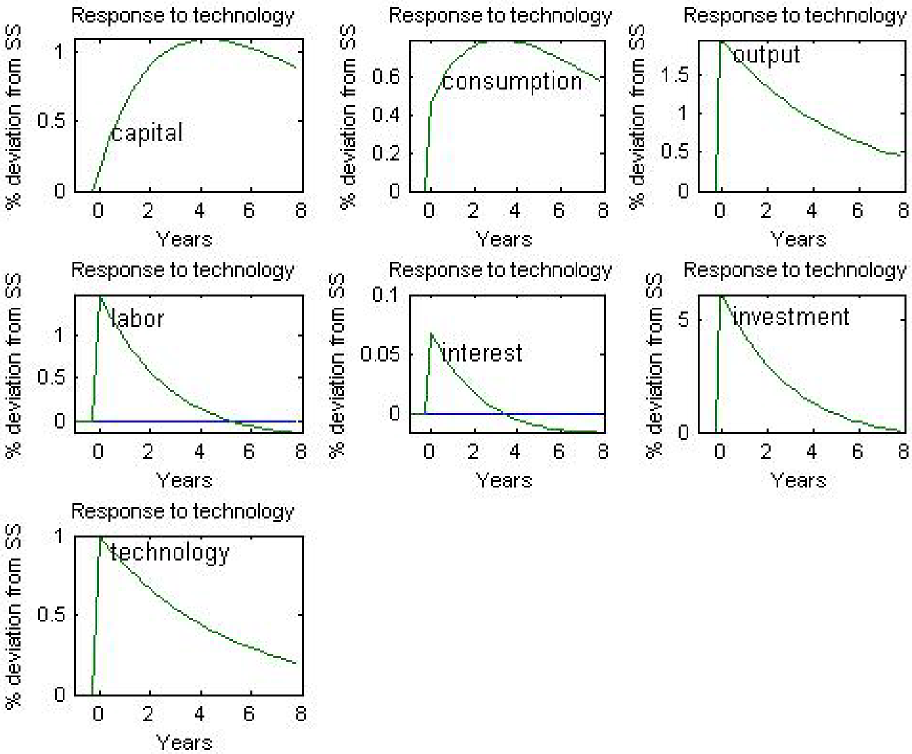
$a = 1/\bar{C} * (1 - \rho) * \bar{Y}/\bar{N} = 2.58$ (previously it was 2.58)

Interpretation: With an increase in technology, output increases. Moreover, since $\bar{N}$ is fixed in the model, marginal product of capital increases with an increase in technology. Hence, it is optimal to invest more in capital. The steady state of capital increases, so does the steady state of investment required to keep the capital the same at the steady state. Since output increases, consumption increases as well. In general, we can interpret these changes as being the results of a more productive economy

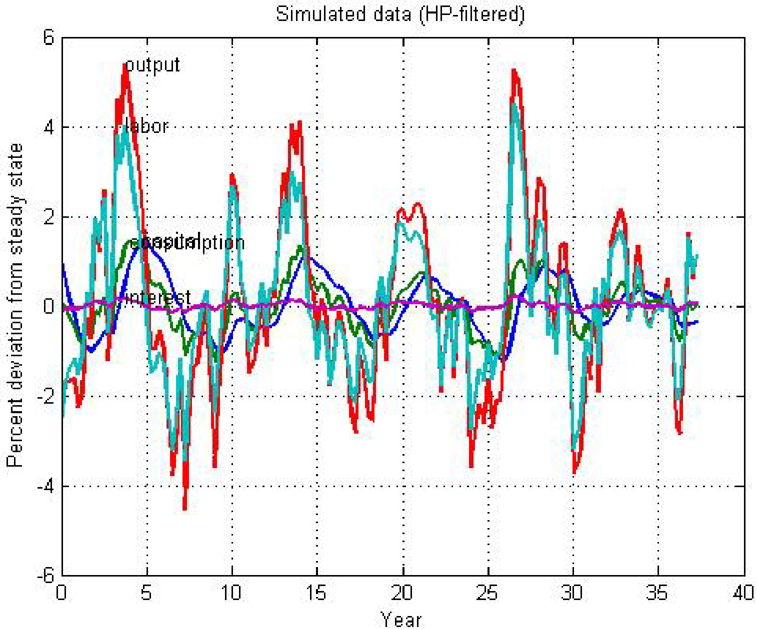
Note: What we have just seen is not directly related to the Business Cycles, as we have looked for the effect of a permanent increase in productivity and Business Cycles arise from temporary shocks. However, analyzing the effect of temporary shocks requires the use of computer programs; hence, we used permanent shocks to get an inference about the effect of a temporary shock. If you compare the changes in the variables on following graph and on the previous slide, you would see that they are qualitatively the same



In separate graphs:



Finally, the figure below shows the data simulated from the model when the model is supplied with productiviy shocks continuously. It shows how the variables moves compared to each other. If the model is correct, the measurements from this data are expected to match the volatility, autocorrelation, and correlation of the variables in the real data (on the next slide)



The characteristics of the real data is on the first two columns, and that of the model which is shown on the previous graph is on the last two columns

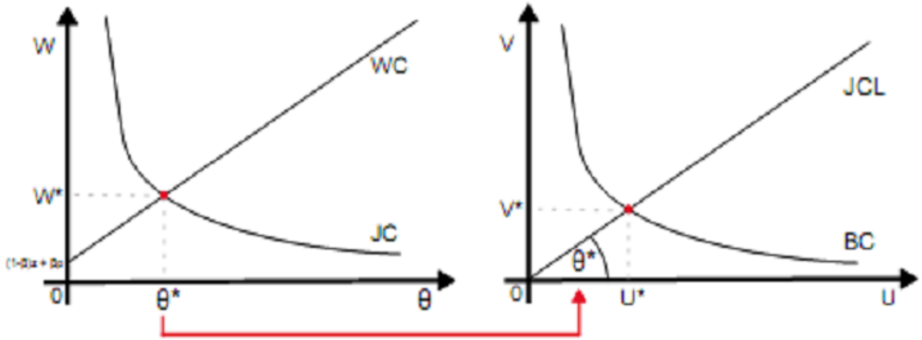
Standard deviations in percent (a) and correlations with output economies.

	Quarterly U.S. time series[a] (55,3–84,1)		Economy with divisible labor[b]	
Series	(a)	(b)	(a)	(b)
Output	1.76	1.00	1.35 (0.16)	1.00 (0.00)
Consumption	1.29	0.85	0.42 (0.06)	0.89 (0.03)
Investment	8.60	0.92	4.24 (0.51)	0.99 (0.00)
Capital stock	0.63	0.04	0.36 (0.07)	0.06 (0.07)
Hours	1.66	0.76	0.70 (0.08)	0.98 (0.01)
Productivity	1.18	0.42	0.68 (0.08)	0.98 (0.01)

Problem 2 (from lecture notes): Recall the three equations (Beveridge Curve, Job Creation Condition and Wage Condition) for the equilibrium conditions of the Search and Matching Model

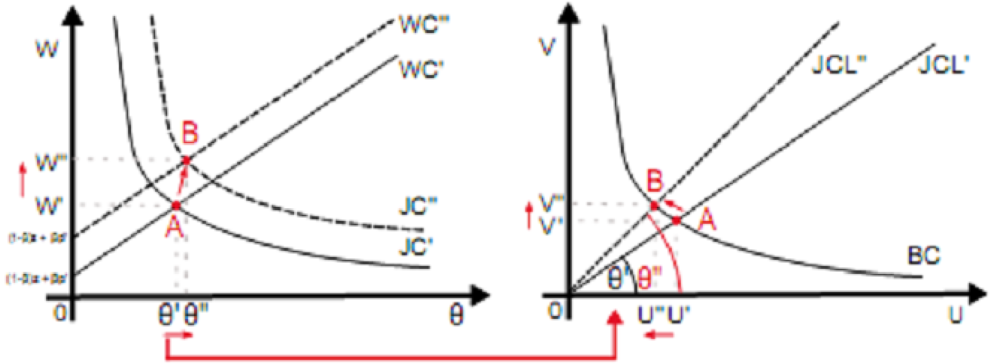
$$BC : u = \frac{\lambda}{\lambda + \theta q(\theta)} \qquad JC : p - w = \frac{(r+\lambda)pc}{q(\theta)} \qquad WC : w = (1-\beta)z + \beta p(1 + c\theta)$$

Analyze the effect of an increase in productivity (from p' to p'') graphically on w, u, v. Also give an economic interpretation when you move the graphs



$$BC : u = \frac{\lambda}{\lambda + \theta q(\theta)} \qquad JC : p - w = \frac{(r+\lambda)pc}{q(\theta)} \qquad WC : w = (1-\beta)z + \beta p(1+c\theta)$$

Solution:



· The rise in productivity (p) shifts Wage Equation (WC) upwards because the more productive workers demand higher wage for any level of θ (if this would have been the only effect, w would increase and $\theta(= v/u)$ would decrease (because when workers demand higher wages firms open less vacancies and unemployment increases)

· But the rise in productivity (p) also shifts Job Creation Equation (JC) rightward because for any level of w, more productive workers give incentive to employers for opening new job positions (vacancies). So θ increases

· As a result: the wage w definitely rises (as it is seen on the left figure)

· Labor market tightness (θ) may rise or not depending on the value of β and c (depending on the intersection of new JC and WC curves)

If θ rises: JCL rotates counterclockwise:Vacancies (v) rise, unemployment (u) falls

If θ declines: JCL rotates clockwise:Vacancies (v) fall, unemployment (u) rises

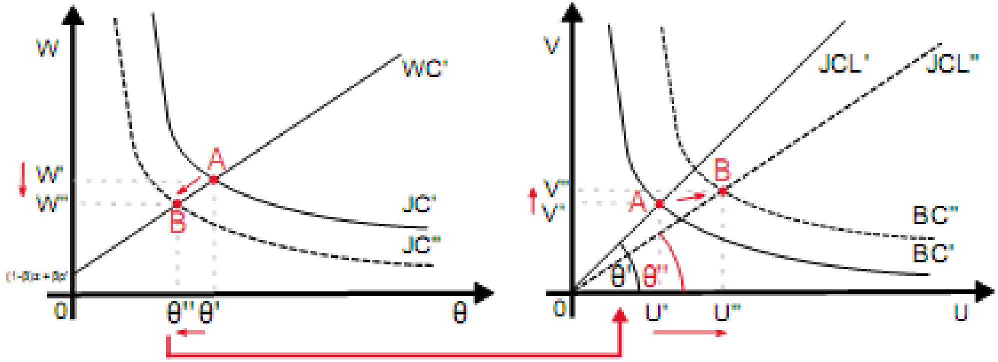
Problem 3: Recall the three equations (Beveridge Curve, Job Creation Condition and Wage Condition) for the equilibrium conditions of the Search and Matching Model

$$BC : u = \frac{\lambda}{\lambda + \theta q(\theta)} \qquad JC : p - w = \frac{(r+\lambda)pc}{q(\theta)} \qquad WC : w = (1-\beta)z + \beta p(1 + c\theta)$$

Analyze the effect of an increase in the job separation rate (λ) graphically on w, u, v. Also given an economic interpretation when you move the line and curves

$$BC: u = \frac{\lambda}{\lambda + \theta q(\theta)} \qquad JC: p - w = \frac{(r+\lambda)pc}{q(\theta)} \qquad WC: w = (1-\beta)z + \beta p(1 + c\theta)$$

Solution:



· The increase in the job separation rate (λ) lowers the value of occupied jobs for employers. Hence, in equilbrium, for any level of w, they open less vacancies. Therefore, Job Creation Equation (JC) shifts leftward. In equilibrium, both w and $\theta(= v/u)$ falls (v decreases u increases)

· Lower θ rotates JCL curve clockwise, as it is seen on the right figure. However, BC curve is also affected from the change in the job separation rate (λ). This is because the higher the λ, the more unemployment and vacancies model produces. Hence, BC curve shifts rightward

- At the equilibrium point B, we have more unemployment for sure
- Whether v increases or not depends on the model parameters