1 Literature Review on Structural Breaks

1.1 Unit Root vs Structural Breaks

According to Perron (2005), there is an "intricate play" between unit root and structural breaks. Most tests that attempt to distinguish between a unit root and a (trend) stationary process will favor the unit root model when the true process is subject to structural changes but is otherwise (trend) stationary within regimes specified by the break dates. Also, most tests trying to assess whether structural change is present will reject the null hypothesis of no structural change when the process has a unit root component but with constant model parameters. Accordingly there is voluminous literature on testing unit root under structural break(s). These tests also give break dates as by-product, but they are not as efficient as the break estimators. The details follow.

Perron (1989) tests null hypothesis of unit root under the assumption of known (*exogenous, pre-tested*) break date in both null and alternative hypothesis. Later Christiano (1992) criticizes Perron's known date assumption as "data mining". He argues that the data based procedures are typically used to determine the most likely location of the break, i.e. by pre-test examination of the data, and this approach invalidates the distribution theory underlying conventional testing. Zivot and Andrews (1992) and Perron (1997) proposed determining the break point "endogenously" from the data. However, these endogenous tests were criticized for their treatment of breaks under the null hypothesis. They do not allow for break(s) under the null hypothesis of unit root and derive their critical values accordingly. So they exclude the possibility that there may be unit root process with break, and under this case, these tests declares data as stationary with breaks. So it seems literature on this subject come to the point that uses Lee and Strazicich (2003), (2004) approach of minimum Lagrange Multiplier (LM) tests, one allowing two-breaks in time series data and the other allows one. While testing for a unit root, they both estimate break date(s) endogenously from the data, and also allow break(s) both under null and alternative hypothesis. By simulation exercises they show that their test outperforms the existing ones.

1.2 Structural Break Estimation and Testing

Structural break tests can be divided into three categories. Chow test is used within the first category. It tests whether the series has a break *in the tested date.* The tests in the second category *look for the presence of a break* in the series, which may exist in any time within the sample period. Some tests in this category also reveals the most possible break date as by-product. The tests in the last category are *estimators indeed,* *they first estimate the "unknown" date of the break, then test it.*

For any type of break, the date of the break, if it exists, is unknown so that falls into the third category. But to understand the basics of the structural break estimators that are used to find unknown break dates and test them, it is better to start with the Chow Test. It is because "unknown" date estimators that are using more complicated tests basically rests on the same principles with this test.

Chow test looks for; whether splitting data from the possible break point and estimating two generated sub-sample separately by least square gives significantly better fitting than using whole sample at once; if the answer is yes, the null hypothesis of no break is rejected. The resulting statistics would be; F-statistics, log likelihood ratio or Wald statistic.

Given this info on the Chow test, now we will write about the tests (estimators) fall into third category, and mention the tests in the second category when it is necessary. However, as there can be more than one break in the data, the estimators can further be divided into two categories; single break estimators and multiple break estimators. Actually it is theoretically proven that consistency for the break date estimates is satisfied for single break estimators Bai (1997b) and Bai and Perron (1998) even if there exist more than one break in the data, which works in a way of first finding one break in the data, and then splitting the data from there and search for new ones on the new samples[1]. Yet, multiple break estimators is used to get more precise estimates, i.e. to find smaller confidence intervals around the breaks, and also to increase the rate of convergence to the break dates that also increases efficiency in the estimation of parameter values subject to the structural change. However, since efficiency is not always the concern of applied economists, we will review the literature for both type estimators. Finally, Multi-Equations Systems is used to get more precise estimates for any type of estimator.

Single Break Estimators: For the "unknown" break date, Quandt (1958, 1960) proposed a likelihood ratio test statistics for unknown change point, called *Supremum (Max)-Test* , while Andrews (1993) supplied analogous Wald and Lagrange Multiplier test statistics for it. Then Andrews and Ploberger (1994) developed *Exponential* (LR, Wald and LM) and *Average* (LR, Wald and LM) tests. These tests are calculated by using individual Chow Statistics for each date of the data except from some trimmed portion from both ends of it. While Sup test is calculated for and finds the date that

[1]This is due to the following result. When estimating a single break model in the presence of multiple breaks, the estimate of the break fraction will converge to one of the true break fractions, the one that is dominant in the sense that taking it into account allows the greatest reduction in the sum of squared residuals (in the case of two breaks that are equally dominant, the estimate will converge with probability 1/2 to either break).

maximizes Chow Statistics, the most possible break point, Ave and Exp tests use all the Chow statistics values and are only informative about existence of the break but not its date. The deficiency of Supremum test is, however, it has power only if one break occurs under alternative hypothesis and is valid as long as residuals from regression follows i.i.d., they do not show heterogeneity before and after the break, as it is also necessary condition for Chow test. Heteroscedasticity and autocorrelation robust version of this test (also called Quandt Likelihood Ratio or Andrews-Quandt statistics, which is the estimator used most commonly in this literature) can be used even though it stills give the most possible break date (it is so because of the small sample properties), it also strongly suffers from finding large confidence intervals around break date.

Multiple Break Estimators: Perron and Qu (2006), following the work of Bai and Perron (1998) & (2003), first define minimum segment length (in proportion to the total data), then given this constraint, search for optimal partition of all possible segments of data to obtain global minimizers of the sum of squared residuals. By this way, they obtain the location of breaks minimizing their objective function for any possible number of breaks. Then they sequentially test for whether additional break date significantly reduces sum of squared errors. Their methodology inherits both pure or partial structural change models (the latter means only several variables are subjected to change). Though this method consistently find the break dates, there is Perron's (2005) himself's comment on this procedure that "the fact that the method of estimation is based on the least-squares principle implies that, even if changes in the variance of error terms are allowed, provided they occur at the same dates as the breaks in the parameters of the regression, such changes are not exploited to increase the precision of the break date estimators. This is due to the fact that the least-squares method imposes equal weights on all residuals. Allowing different weights, as needed when accounting for changes in variance, requires adopting a quasi-likelihood framework".

Finally Perron and Qu (2007) brings what we think of as novel approach to structural change analysis. They use multiple equation model and quasi-likelihood framework based on normal errors. They first define minimum segment length of the data that could be separated with breaks, and then given this constraint, search for the optimal partition of all possible segments of data the model fits, while the objective function tried to be maximized is quasi-likelihood one. Their methodology is able to find the break even it exists only in one of the used equations[2]. The reason for using multiple number of equations then is that, with their words, "there can be an increase in the precision of the estimates as long as the correlation between error terms of equations are different from zero. Intuition is; a poorly estimated break in one regression affects the likelihood function through the residual variance of that equation but also via the correlation with

[2]We confirmed their result by running simulations

the rest of the regressions. Hence, by including ancillary equations without breaks, additional forces are in play to better pinpoint the break dates". Just like using SUR model is more efficient than the OLS under similar corresponding circumstances. Finally, in their work the error process is allowed to be autocorrelated as well as conditionally heteroskedastic.

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